9. [9 points] Consider the region $R$ bounded by the curves $y = x^2$, $y = x + 2$ and the $y$-axis, where $x$ and $y$ are measured in meters.

![Graph showing the region R bounded by the curves y = x^2 and y = x + 2.]

**a. [5 points]** Let $T$ be the solid obtained by rotating the region $R$ about the $x$-axis. Find a formula involving definite integrals that computes the volume of $T$.

**Solution:** Using washers: $V = \int_0^2 \pi [(x + 2)^2 - x^4] dx$.

Using shells: $V = \int_0^2 2\pi y \sqrt{y} dy + \int_2^4 2\pi y (\sqrt{y} - (y - 2)) dy$.

**b. [2 points]** The mass density of the solid $T$ is given by the function $\delta(x) = 2 - \sqrt{x}$ kg per m$^3$. Find a formula involving definite integrals that computes the mass of $T$.

**Solution:** Since the density depends on the variable $x$, you need to take slices perpendicular to the $x$-axis. Hence

$$m = \int_0^2 (2 - \sqrt{x}) \pi [(x + 2)^2 - x^4] dx.$$

**c. [2 points]** Find a formula involving definite integrals that computes the value of $\bar{x}$, the $x$ coordinate of the center of mass of the solid $T$.

**Solution:**

$$\bar{x} = \frac{\int_0^2 x(2 - \sqrt{x}) \pi [(x + 2)^2 - x^4] dx}{\int_0^2 (2 - \sqrt{x}) \pi [(x + 2)^2 - x^4] dx}.$$