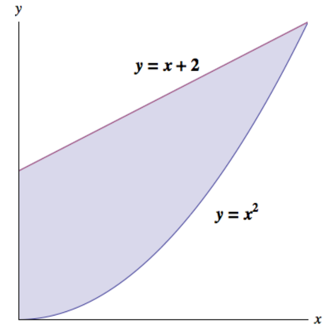
9. [9 points] Consider the region R bounded by the curves $y = x^2$, $y = x + 2$ and the y-axis, where x and y are measured in meters.



a. [5 points] Let T be the solid obtained by rotating the region R about the x-axis. Find a formula involving definite integrals that computes the volume of T.

Solution: Using washers: $V = \int_0^2 \pi[(x+2)^2 - x^4]dx$.

Using shells: $V = \int_0^2 2\pi y\sqrt{y}dy + \int_2^4 2\pi y(\sqrt{y} - (y-2))dy$

b. [2 points] The mass density of the solid T is given by the function $\delta(x) = 2 - \sqrt{x}$ kg per m^3. Find a formula involving definite integrals that computes the mass of T.

Solution: Since the density depends on the variable x, you need to take slices perpendicular to the x-axis. Hence

$$m = \int_0^2 (2-\sqrt{x})\pi[(x+2)^2 - x^4]dx.$$

c. [2 points] Find a formula involving definite integrals that computes the value of $\bar{x}$, the x coordinate of the center of mass of the solid T.

Solution:

$$\frac{\int_0^2 x(2-\sqrt{x})\pi[(x+2)^2 - x^4]dx}{\int_0^2 (2-\sqrt{x})\pi[(x+2)^2 - x^4]dx}.$$