

1. [7 points] The table below gives values of a function,  $f(x)$ , at several points.

$x$	4	5	6	7	8
$f(x)$	3	5	4	1	2

- a. [3 points] Estimate the integral  $\int_4^8 f(x)dx$  using Mid(2). Be sure to write out all the terms of your sum.

*Solution:*

$$\text{Mid}(2) = 2(f(5) + f(7)) = 2(5 + 1) = 12.$$

- b. [4 points] Simplify the integral  $\int_{\ln(4)}^{\ln(7)} e^x f(e^x) dx$  and estimate the resulting integral using Trap(3). Be sure to show how you simplified the integral and to write out all the terms of your sum.

*Solution:*

Let  $u = e^x$  then  $du = e^x dx$ . Changing the bounds of integration upper bound =  $e^{\ln(7)} = 7$ ,

lower bound =  $e^{\ln(4)} = 4$ . Thus  $\int_{\ln(4)}^{\ln(7)} e^x f(e^x) dx = \int_4^7 f(u) du$ .

$$\text{Trap}(3) = \frac{1}{2}(\text{Left}(3) + \text{Right}(3)) = \frac{1}{2}f(4) + f(5) + f(6) + \frac{1}{2}f(7) = 11.$$

2. [5 points] Suppose that  $g(x) = w(x)v(x)$  where the functions  $w(x)$  and  $v(x)$  are both positive, decreasing and concave down on the interval  $[0, 1]$ .

- a. [2 points] Write the derivatives  $g'(x)$  and  $g''(x)$  in terms of  $w(x)$ ,  $v(x)$ , and their derivatives.

*Solution:*

$$g'(x) = w'(x)v(x) + w(x)v'(x)$$

$$g''(x) = w''(x)v(x) + 2w'(x)v'(x) + w(x)v''(x)$$

- b. [3 points] Circle the method(s) that will ALWAYS UNDERESTIMATE the integral  $\int_0^1 g(x)dx$ .

Left

Right

Mid

Trap