**1**. [7 points] The table below gives values of a function, $f(x)$, at several points.

| $x$ | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|
| $f(x)$ | 3 | 5 | 4 | 1 | 2 |

**a**. [3 points] Estimate the integral $\int_4^8 f(x)dx$ using Mid(2). Be sure to write out all the terms of your sum.

*Solution:*
Mid(2) $= 2(f(5) + f(7)) = 2(5 + 1) = 12$.

**b**. [4 points] Simplify the integral $\displaystyle\int_{\ln(4)}^{\ln(7)} e^x f(e^x)dx$ and estimate the resulting integral using Trap(3). Be sure to show how you simplified the integral and to write out all the terms of your sum.

*Solution:*
Let $u = e^x$ then $du = e^x dx$. Changing the bounds of integration upper bound$= e^{\ln(7)} = 7$, lower bound$= e^{\ln(4)} = 4$. Thus $\displaystyle\int_{\ln(4)}^{\ln(7)} e^x f(e^x)dx = \int_4^7 f(u)du$.
Trap(3) $= \frac{1}{2}$(Left(3)+Right(3)) $= \frac{1}{2}f(4) + f(5) + f(6) + \frac{1}{2}f(7) = 11$.

**2**. [5 points] Suppose that $g(x) = w(x)v(x)$ where the functions $w(x)$ and $v(x)$ are both positive, decreasing and concave down on the interval $[0, 1]$.

**a**. [2 points] Write the derivatives $g'(x)$ and $g''(x)$ in terms of $w(x)$, $v(x)$, and their derivatives.

*Solution:*
$g'(x) = w'(x)v(x) + w(x)v'(x)$
$g''(x) = w''(x)v(x) + 2w'(x)v'(x) + w(x)v''(x)$

**b**. [3 points] Circle the method(s) that will ALWAYS UNDERESTIMATE the integral $\int_0^1 g(x)dx$.

Left  **Right**  Mid  Trap