7. [8 points] Alyssa Edwards wants to play a prank on Coco Montrese by spilling a bucket of orange cheese powder on her. To do this Alyssa lifts the bucket at a constant speed from the ground to a height of 10 meters. Unfortunately the bucket has a small hole and the cheese begins leaking out at a constant rate as soon as the bucket leaves the ground. The bucket initially weighs 10kg and when it reaches a height of 10 meters it only weighs 5kg. Recall the gravitational constant is $g = 9.8\text{m/s}^2$.

a. [3 points] Write an expression giving the mass of the bucket $m(h)$ when the bucket is h meters above the ground.

Solution: The bucket is being lifted and is leaking at a constant rate. Therefore the mass of the bucket at height h will be a linear function. $m(h) = 5 + (\frac{10-h}{2}) = 10 - \frac{h}{2}$

b. [5 points] How much work is required to lift the bucket from the ground to a height of 10 meters? Include units.

Solution: The force on the bucket at height h is $gm(h)$. Therefore the work is $\int_0^{10} gm(h)dh = \int_0^{10} g(10 - \frac{h}{2})dh = 75g = 735$ joules.