

4. [14 points] The function

$$f(x) = \sin(\sqrt{x})$$

does not have an antiderivative that can be written in terms of elementary functions. However, we can use the second fundamental theorem of calculus to construct an antiderivative for f . We define an antiderivative F of f by

$$F(x) = \int_0^x \sin(\sqrt{t}) dt.$$

- a. [2 points] The concavity of F does not change on the interval $(0, \frac{\pi^2}{4})$. Determine the concavity of F on $(0, \frac{\pi^2}{4})$ and circle one of the options below. No justification is needed.

Concave Up

Concave Down

Neither

- b. [2 points] Using the blanks provided, order from least to greatest

$$F\left(\frac{\pi^2}{4}\right), \quad \text{LEFT}(100), \quad \text{RIGHT}(100), \quad \text{MID}(100), \quad \text{TRAP}(100),$$

where all the approximations are of the definite integral given by $F\left(\frac{\pi^2}{4}\right)$. No justification is needed.

_____ \leq _____ \leq _____ \leq _____ \leq _____

- c. [4 points] Write out, but do not compute, MID(3) to approximate $F\left(\frac{\pi^2}{4}\right)$.

- d. [4 points] Write out, but do not compute, TRAP(3) to approximate $F\left(\frac{\pi^2}{4}\right)$.

- e. [2 points] If you want to approximate $F\left(\frac{\pi^2}{4}\right)$ using right and left sums, what is the smallest number of subdivisions, n , you would have to use to guarantee that the difference between LEFT(n) and RIGHT(n) is less than or equal to 0.005?