

7. [16 points] In each part, circle “True” if the statement is always true and circle “False” otherwise. No justification is necessary. Any unclear markings will be marked incorrect.

a. [8 points] Suppose $g(x)$ is a positive function, defined for all real numbers x , with continuous first derivative.

$$(1) \int_0^7 xg(x^2) dx = \int_0^7 g(u) du.$$

True False

$$(2) \int_0^7 xg(x^2) dx = \frac{1}{2} \int_0^{49} g(t) dt.$$

True False

$$(3) \int_0^7 xg(x^2) dx = 7g(49) - \int_0^7 g(x^2) dx.$$

True False

$$(4) \int_0^7 xg(x^2) dx = \frac{49}{2}g(49) - \int_0^7 x^3g'(x^2) dx.$$

True False

b. [8 points] Suppose $h(y)$ is the density, in grams per cm, of a thin rod of length 10 cm, y cm from one end. Suppose the rod has mass M .

$$(1) \int_0^5 h(y) dy = \frac{M}{2}.$$

True False

$$(2) \text{ The center of mass of the rod is } \int_0^{10} yh(y) dy.$$

True False

$$(3) \text{ If } h(y) \text{ is a constant function, then } h(y) = \frac{M}{10}.$$

True False

$$(4) \text{ The average value of } h(y) \text{ on } [0, 10] \text{ is } \frac{M}{10}.$$

True False