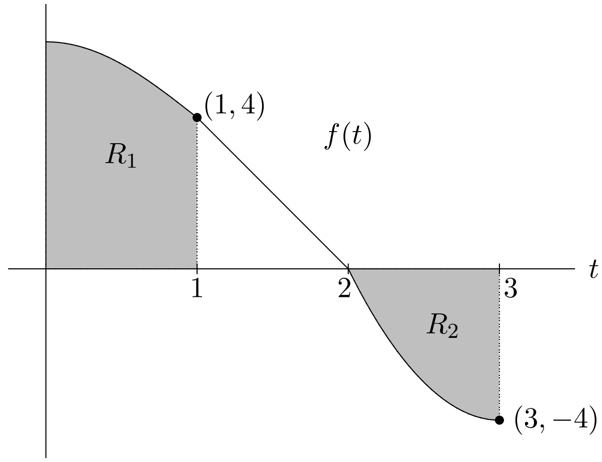
1. [14 points] While you are trying to fill your old bucket with water, it begins to leak. Suppose the continuous function $f(t)$ is the rate of change of the volume of water in the bucket, in gallons per minute, t minutes after it begins to leak. A graph of $f(t)$ for $0 \le t \le 3$ is shown below. The function $f(t)$ is linear for $1 \le t \le 2$. The region R_1 has area 5.8, and the region R_2 has area 3. There are 7 gallons of water in the bucket at $t = 1$.



a. [5 points] Write an expression involving integrals for $A(t)$, the volume of water in the bucket, in gallons, t minutes after the bucket began to leak where $0 \le t \le 3$. Your expression may contain the function f.

Solution: $A(t) = 7 + \int_1^t f(x)dx$

b. [2 points] How much water was in the bucket when it began to leak? How much water was in the bucket 3 minutes after it began to leak? Fill in the blanks below.

Solution:
There were 1.2 gallons of water in the bucket when it began to leak.
There were 6 gallons of water in the bucket 3 minutes after it began to leak.

c. [3 points] Write an expression involving an integral for the average rate of change of the amount of water in the bucket during the first three minutes after it began to leak, and find the value of your expression, including units.

Solution: $\frac{1}{3-0}\int_0^3 f(t)dt = \frac{1}{3}[A(3) - A(0)] = \frac{4.8}{3}$ gal/min.

d. [4 points] For $t \geq 3$, suppose $f(t)$ is linear with slope 1, but is only defined until the time when the bucket is empty. For what value of t is the bucket empty? (Remember that f is continuous as specified above).

Solution: For $t \geq 3$, $f(t)$ is linear with slope 1 and passes through the point $(3, -4)$. So, $f(t) = t - 7$ for $t \geq 3$ until the time when the bucket is empty. To find the t value where the bucket is empty, we can solve $\int_3^t x - 7dx = -6$ to get $t^2 - 14t + 45 = 0$. We can factor the quadratic polynomial in t to get that $t = 5$ or $t = 9$. Then $f(t)$ will be defined until $t = 5$.