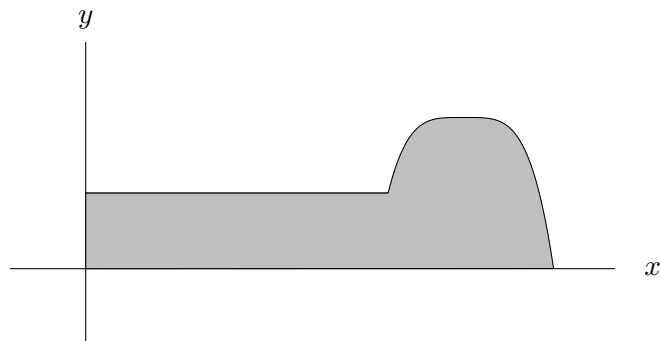**2**. [13 points] Fred is designing a plastic bowl for his dog, Fido. Fred makes the bowl in the shape of a solid formed by rotating a region in the xy-plane around the y-axis. The region, shaded in the figure below, is bounded by the x-axis, the y-axis, the line $y = 1$ for $0 \leq x \leq 4$, and the curve $y = -(x - 5)^4 + 2$ for $4 \leq x \leq 2^{1/4} + 5$. Assume the units of x and y are inches.

a. [7 points] Write an expression involving one or more integrals which gives the volume of plastic needed to make Fido's bowl. What are the units of your expression?

> *Solution:*   Using the cylindrical shell method, we have that the volume of plastic needed to make Fido's bowl is given by $\displaystyle\int_0^4 2\pi x\, dx + \int_4^{5+2^{\frac{1}{4}}} 2\pi x(2 - (x - 5)^4)\, dx$.
>
> Using the washer method, we have that the volume of plastic needed to make Fido's bowl is given by $\displaystyle\pi \int_0^1 (5 + (2 - y)^{1/4})^2\, dy + \pi \int_1^2 (5 + (2 - y)^{1/4})^2 - (5 - (2 - y)^{1/4})^2\, dy$.
>
> The units for either expression are in^3.

b. [6 points] Fred wants to wrap a ribbon around the bowl before he gives it to Fido as a gift. The figure below depicts the cross section of the bowl obtained by cutting it in half across its diameter. The thick solid curve is the ribbon running around this cross section, and the dotted curve is the outline of the cross section which is not in contact with the ribbon. Write an expression involving one or more integrals which gives the length of the thick solid curve in the figure (the length of ribbon Fred needs to wrap the bowl).

> *Solution:*   The length of ribbon Fred needs to wrap the bowl is given by
>
> $10 + 2(2^{\frac{1}{4}} + 5) + 2\displaystyle\int_5^{5+2^{\frac{1}{4}}} \sqrt{1 + 16(x - 5)^6}\, dx$.