3. [13 points] Use the table and the fact that

$$\int_0^{10} f(t)dt = 350$$

to evaluate the definite integrals below exactly (i.e., no decimal approximations). Assume $f'(t)$ is continuous and does not change sign between any consecutive t-values in the table.

t	0	10	20	30	40	50	60
$f(t)$	0	70	e^5	e^3	0	$\pi/2$	π

a. [4 points] $\int_0^{10} tf'(t)dt$

Solution:

$$\begin{aligned}\int_0^{10} tf'(t)dt &= tf(t)|_0^{10} - \int_0^{10} f(t)dt \\ &= 10f(10) - \int_0^{10} f(t)dt \\ &= 700 - 350 \\ &= 350.\end{aligned}$$

b. [4 points] $\int_{20}^{30} \frac{f'(t)}{f(t)}dt$

Solution:

$$\begin{aligned}\int_{20}^{30} \frac{f'(t)}{f(t)}dt &= \int_{f(20)}^{f(30)} \frac{1}{u}du \\ &= \ln|u||_{f(20)}^{f(30)} \\ &= \ln|f(30)| - \ln|f(20)| \\ &= 3-5 \\ &= -2.\end{aligned}$$

c. [5 points] $\int_{50}^{60} f(t)f'(t)\sin(f(t))dt$

Solution:

$$\begin{aligned}\int_{50}^{60} f(t)f'(t)\sin(f(t))dt &= \int_{f(50)}^{f(60)} w\sin(w)dw \\ &= -w\cos(w)|_{f(50)}^{f(60)} + \int_{f(50)}^{f(60)} \cos(w)dw \\ &= -\pi\cos(\pi) + \int_{\pi/2}^{\pi} \cos(w)dw \\ &= \pi - 1\end{aligned}$$