3. [13 points] Use the table and the fact that
\[ \int_0^{10} f(t) dt = 350 \]

To evaluate the definite integrals below exactly (i.e., no decimal approximations). Assume \( f'(t) \) is continuous and does not change sign between any consecutive \( t \)-values in the table.

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(t) )</td>
<td>0</td>
<td>70</td>
<td>( e^5 )</td>
<td>( e^3 )</td>
<td>0</td>
<td>( \pi/2 )</td>
<td>( \pi )</td>
</tr>
</tbody>
</table>

a. [4 points] \( \int_0^{10} tf'(t) dt \)

\[
\int_0^{10} tf'(t) dt = tf(t)|_{t=0}^{t=10} - \int_0^{10} f(t) dt
\]
\[ = 10f(10) - \int_0^{10} f(t) dt
\]
\[ = 700 - 350 = 350. \]

b. [4 points] \( \int_{20}^{30} \frac{f'(t)}{f(t)} dt \)

\[
\int_{20}^{30} \frac{f'(t)}{f(t)} dt = \int_{f(20)}^{f(30)} \frac{1}{u} du
\]
\[ = \ln |u| \bigg|_{f(20)}^{f(30)}
\]
\[ = \ln |f(30)| - \ln |f(20)|
\]
\[ = 3 - 5 = -2. \]

c. [5 points] \( \int_{50}^{60} f(t)f'(t) \sin(f(t)) dt \)

\[
\int_{50}^{60} f(t)f'(t) \sin(f(t)) dt = \int_{f(50)}^{f(60)} w \sin(w) dw
\]
\[ = -w \cos(w) \bigg|_{f(50)}^{f(60)} + \int_{f(50)}^{f(60)} \cos(w) dw
\]
\[ = -\pi \cos(\pi) + \int_{\pi/2}^{\pi} \cos(w) dw
\]
\[ = \pi - 1. \]