4. [14 points] The function

$$f(x) = \sin(\sqrt{x})$$

does not have an antiderivative that can be written in terms of elementary functions. However, we can use the second fundamental theorem of calculus to construct an antiderivative for f. We define an antiderivative F of f by

$$F(x) = \int_0^x \sin(\sqrt{t})dt.$$

a. [2 points] The concavity of F does not change on the interval $\left(0, \frac{\pi^2}{4}\right)$. Determine the concavity of F on $\left(0, \frac{\pi^2}{4}\right)$ and circle one of the options below. No justification is needed.

Solution:

Concave Up **Concave Down** **Neither**

b. [2 points] Using the blanks provided, order from least to greatest

$$F\left(\frac{\pi^2}{4}\right), \quad \text{LEFT}(100), \quad \text{RIGHT}(100), \quad \text{MID}(100), \quad \text{TRAP}(100),$$

where all the approximations are of the definite integral given by $F\left(\frac{\pi^2}{4}\right)$. No justification is needed.

Solution: LEFT (100) $\leq$ TRAP (100) $\leq$ $F\left(\frac{\pi^2}{4}\right)$ $\leq$ MID (100) $\leq$ RIGHT (100)

c. [4 points] Write out, but do not compute, MID (3) to approximate $F\left(\frac{\pi^2}{4}\right)$.

Solution: $\text{MID}(3) = \left[\sin\left(\sqrt{\frac{\pi^2}{24}}\right) + \sin\left(\sqrt{\frac{3\pi^2}{24}}\right) + \sin\left(\sqrt{\frac{5\pi^2}{24}}\right)\right]\left(\frac{\pi^2}{12}\right)$

d. [4 points] Write out, but do not compute, TRAP (3) to approximate $F\left(\frac{\pi^2}{4}\right)$.

Solution: We have $\text{LEFT}(3) = \left(\sin(0) + \sin\left(\sqrt{\frac{\pi^2}{12}}\right) + \sin\left(\sqrt{\frac{2\pi^2}{12}}\right)\right)\left(\frac{\pi^2}{12}\right)$,
and $\text{RIGHT}(3) = \left(\sin\left(\sqrt{\frac{\pi^2}{12}}\right) + \sin\left(\sqrt{\frac{2\pi^2}{12}}\right) + \sin\left(\sqrt{\frac{3\pi^2}{12}}\right)\right)\left(\frac{\pi^2}{12}\right)$.
Then $\text{TRAP}(3) = \frac{\text{LEFT}(3)+\text{RIGHT}(3)}{2}$.

e. [2 points] If you want to approximate $F\left(\frac{\pi^2}{4}\right)$ using right and left sums, what is the smallest number of subdivisions, n, you would have to use to guarantee that the difference between LEFT(n) and RIGHT(n) is less than or equal to 0.005?

Solution:

$$\text{RIGHT}\,(n) - \text{LEFT}\,(n) < 0.005 \iff \left(\sin\left(\sqrt{\frac{\pi^2}{4}}\right) - \sin(0)\right)\frac{\frac{\pi^2}{4} - 0}{n} < 0.005$$

$$\iff n > \frac{\pi^2}{0.02} \approx 493.$$

Since we need an integer number of subdivisions, we take $n = 494$.