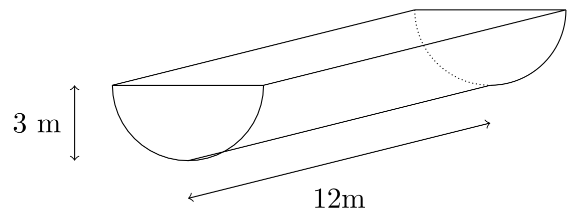
5. [11 points] Robber baron and philanthropist Calvin Currency is making a large cash donation in $100 bills. Before making the donation, he decides to fill an empty pool with the money. The pool is a half cylinder with radius 3 meters and length 12 meters as shown below. After an afternoon of diving into his pool of money and swimming around, the distribution of bills in the pool becomes nonuniform and so the density of money in the pool is given by

$$\delta(y) = 30,000\sqrt{\frac{10}{\pi}}e^{-y^2},$$

measured in bills per m^3, where y is height in meters measured from the bottom of the pool. Recall the gravitational constant is $g = 9.8$ m/s^2



a. [5 points] Write a definite integral which gives the volume of the pool.

Solution: The volume of the pool is $\int_0^3 12(2\sqrt{9-(y-3)^2})dy$.

b. [2 points] Write a definite integral which gives the value of the money in the pool, in dollars.

Solution: The value of money in the pool is given by $100\int_0^3 \delta(y)(12)(2\sqrt{9-(y-3)^2})dy$.

c. [4 points] Write a definite integral which gives the amount of work done in lifting the money out of the pool if each bill has mass 0.001 kg.

Solution: The work done in lifting the money out of the pool is given by
$\int_0^3 (0.001)\delta(y)(g)(12)(2\sqrt{9-(y-3)^2})(3-y)dy$.