7. [16 points] In each part, circle "True" if the statement is always true and circle "False" otherwise. No justification is necessary. Any unclear markings will be marked incorrect.

> *Solution:*

> **a.** [8 points] Suppose $g(x)$ is a positive function, defined for all real numbers x, with continuous first derivative.
>
> (1) $\displaystyle\int_0^7 xg(x^2)\,dx = \int_0^7 g(u)\,du.$
>
> True   <u>False</u>
>
> (2) $\displaystyle\int_0^7 xg(x^2)\,dx = \frac{1}{2}\int_0^{49} g(t)\,dt.$
>
> <u>True</u>   False
>
> (3) $\displaystyle\int_0^7 xg(x^2)\,dx = 7g(49) - \int_0^7 g(x^2)\,dx.$
>
> True   <u>False</u>
>
> (4) $\displaystyle\int_0^7 xg(x^2)\,dx = \frac{49}{2}g(49) - \int_0^7 x^3 g'(x^2)\,dx.$
>
> <u>True</u>   False

> **b.** [8 points] Suppose $h(y)$ is the density, in grams per cm, of a thin rod of length 10 cm, y cm from one end. Suppose the rod has mass M.
>
> (1) $\displaystyle\int_0^5 h(y)\,dy = \frac{M}{2}.$
>
> True   <u>False</u>
>
> (2) The center of mass of the rod is $\displaystyle\int_0^{10} yh(y)\,dy.$
>
> True   <u>False</u>
>
> (3) If $h(y)$ is a constant function, then $h(y) = \frac{M}{10}$.
>
> <u>True</u>   False
>
> (4) The average value of $h(y)$ on $[0,10]$ is $\frac{M}{10}$.
>
> <u>True</u>   False