

1. [16 points] At a time  $t$  seconds after a catapult throws a rock, the rock has horizontal velocity  $v(t)$  m/s. Assume  $v(t)$  is monotonic between the values given in the table and does not change concavity.

$t$	0	1	2	3	4	5	6	7	8
$v(t)$	47	34	24	16	10	6	3	1	0

- a. [4 points] Estimate the average horizontal velocity of the rock between  $t = 2$  and  $t = 5$  using the trapezoid rule with 3 subdivisions. Write all the terms in your sum. Include **units**.

*Solution:*

$$\begin{aligned} \frac{\int_2^5 v(t) dt}{5 - 2} &= \frac{Left(3) + Right(3)}{2 \cdot 3} = \frac{(v(2) + v(3) + v(4)) + (v(3) + v(4) + v(5))}{6} = \\ &= \frac{24 + 16 + 10 + 16 + 10 + 6}{6} = \frac{82}{6} = \frac{41}{3} \end{aligned}$$

The average horizontal velocity of the rock is  $41/3$  m/s.

- b. [4 points] Estimate the total horizontal distance the rock traveled using a left Riemann sum with 8 subdivisions. Write all the terms in your sum. Include **units**.

*Solution:*

$$\begin{aligned} \int_0^8 v(t) dt &= Left(8) = v(0) + v(1) + v(2) + v(3) + v(4) + v(5) + v(6) + v(7) = \\ &= 47 + 34 + 24 + 16 + 10 + 6 + 3 + 1 = 141 \end{aligned}$$

The total horizontal distance the rock traveled is approximately 141 meters.

- c. [4 points] Estimate the total horizontal distance the rock traveled using the midpoint rule with 4 subdivisions. Write all the terms in your sum. Include **units**.

*Solution:*

$$\begin{aligned}\int_0^8 v(t) dt &= \text{Mid}(4) = 2(v(1) + v(3) + v(5) + v(7)) = \\ &= 2(34 + 16 + 6 + 1) = 114\end{aligned}$$

The total horizontal distance the rock traveled is approximately 114 meters.

- d. [4 points] A second rock thrown by the catapult traveled horizontally 125 meters. Determine whether the first rock or the second rock traveled farther, or if there is not enough information to decide. Circle your answer. Justify your answer.

*Solution:*

the first rock

the second rock

not enough information

The function  $v(t)$  is concave up since for example

$$-10 = \frac{v(2) - v(1)}{2 - 1} > \frac{v(1) - v(0)}{1 - 0} = -13$$

The trapezoid rule gives  $\text{Trap}(4) = 121$  (or  $\text{Trap}(8) = 117.5$ ). Since  $v(t)$  is concave up, this is an overestimate which means that the first rock traveled at most 121 meters, that is less than the second.