5. [16 points] Suppose that $f(x)$ is a function with the following properties:

- $\displaystyle\int_0^1 f(x)\,dx = -5.$
- $\displaystyle\int_0^3 f'(x)\,dx = 10.$
- The average value of $f(x)$ on $[1, 1.5]$ is -4.
- $\displaystyle\int_2^4 xf'(x)\,dx = 8.$

In addition, a table of values for $f(x)$ is given below.

x	0	1	2	3	4
$f(x)$	-7	-2	-2	m	0

Calculate (a)-(d) **exactly**. Show your work and do not write any decimal approximations.

a. [4 points] $m = 3$

Solution: Using the Fundamental Theorem in $\int_0^3 f'(x)\,dx = 10$ we get $f(3) - f(0) = 10$ which gives $m - (-7) = 10$ so $m = 3$.

b. [4 points] $\displaystyle\int_0^{1.5} f(x)\,dx = -7$

Solution:

$$\int_0^{1.5} f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^{1.5} f(x)\,dx = -5 + 0.5(-4) = -7$$

c. [4 points] $\displaystyle\int_2^4 f(x)\,dx = -4$

Solution: Using integration by parts in $\int_2^4 xf'(x)\,dx = 8$ we get $(4f(4) - 2f(2)) - \int_2^4 f(x)\,dx = 8$ which gives $\int_2^4 f(x)\,dx = 0 - 2(-2) - 8 = -4$.

d. [4 points] $\displaystyle\int_4^{16} f'(\sqrt{x})\,dx = 16$

Solution: Using the substitution $u = \sqrt{x}$ we get

$$\int_4^{16} f'(\sqrt{x})\,dx = \int_2^4 f'(u) \cdot 2u\,du = 2 \cdot 8 = 16$$