6. [10 points] O-guk loves to eat vegetables, especially carrots. Every morning, he eats a bin filled to the top with shredded carrots. The bin is in the shape of a half cylinder and it is pictured below. The density of the carrots at height $h$ m from the bottom of the bin is given by $\delta(h)$ kg/m$^3$.

a. [6 points] To get an idea of how much he eats, write an expression involving integrals that gives the mass of the carrots in the bin. Include units. Don’t compute any integrals.

**Solution:** The volume of a slice at height $h$ of thickness $\Delta h$ is $4 \cdot L \cdot \Delta h$ where $L$ is the width of the slice. Using geometry we have $\left(\frac{L}{2}\right)^2 + (1 - h)^2 = 1^2$ so $L = 2\sqrt{1 - (1 - h)^2}$. The mass of the slice is then $\delta(h) \cdot 4 \cdot 2\sqrt{1 - (1 - h)^2} \cdot \Delta h$ kilograms. The total mass of the carrots in the bin is given by

$$\int_0^1 \delta(h) \cdot 4 \cdot 2\sqrt{1 - (1 - h)^2} \, dh \text{ kgs.}$$

b. [4 points] Write an expression involving integrals that gives the $h$-center of mass of the carrots in the bin. Don’t compute any integrals.

**Solution:**

$$\bar{h} = \frac{\int_0^1 h \cdot \delta(h) \cdot 4 \cdot 2\sqrt{1 - (1 - h)^2} \, dh}{\int_0^1 \delta(h) \cdot 4 \cdot 2\sqrt{1 - (1 - h)^2} \, dh}$$