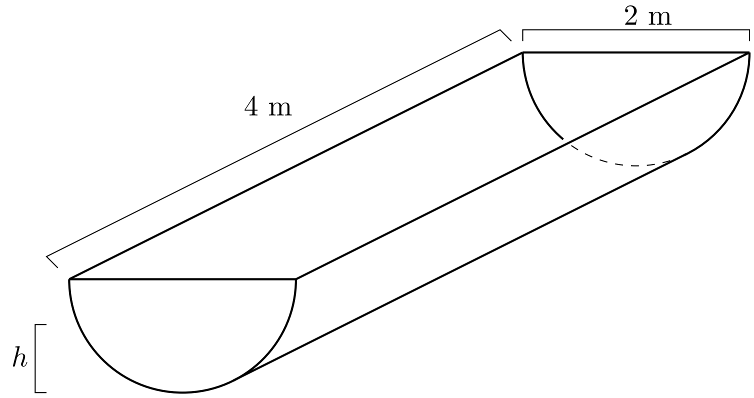
6. [10 points] O-guk loves to eat vegetables, especially carrots. Every morning, he eats a bin filled to the top with shredded carrots. The bin is in the shape of a half cylinder and it is pictured below. The density of the carrots at height h m from the bottom of the bin is given by $\delta(h)$ kg/m^3.



a. [6 points] To get an idea of how much he eats, write an expression involving integrals that gives the mass of the carrots in the bin. Include **units**. Don't compute any integrals.

Solution: The volume of a slice at height h of thickness Δh is $4 \cdot L \cdot \Delta h$ where L is the width of the slice. Using geometry we have $\left(\frac{L}{2}\right)^2 + (1-h)^2 = 1^2$ so $L = 2\sqrt{1-(1-h)^2}$. The mass of the slice is then $\delta(h) \cdot 4 \cdot 2\sqrt{1-(1-h)^2} \cdot \Delta h$ kilograms. The total mass of the carrots in the bin is given by

$$\int_0^1 \delta(h) \cdot 4 \cdot 2\sqrt{1-(1-h)^2}\, dh \quad \text{kgs.}$$

b. [4 points] Write an expression involving integrals that gives the h-center of mass of the carrots in the bin. Don't compute any integrals.

Solution:

$$\bar{h} = \frac{\int_0^1 h \cdot \delta(h) \cdot 4 \cdot 2\sqrt{1-(1-h)^2}\, dh}{\int_0^1 \delta(h) \cdot 4 \cdot 2\sqrt{1-(1-h)^2}\, dh}$$