9. [ 9 points] The tank pictured below has height 2 meters, and the top and bottom are equilateral triangles with sides of length 1 meter. It is filled halfway with hot chocolate. The hot chocolate has uniform density $1325 \mathrm{~kg} / \mathrm{m}^{3}$. The acceleration due to gravity is $9.8 \mathrm{~m} / \mathrm{s}^{2}$. Calculate the work needed to pump all the chocolate to the top of the tank. Show all your work. Give an exact answer. Include units.


Solution: We take a horizontal slice at height $y$ meters from the bottom of the tank. It has mass $1325 \cdot \frac{\sqrt{3}}{4} 1^{2} \Delta y$. We need to move it $2-y$ meters up. Thus, the work needed to pump all the chocolate to the top is

$$
\int_{0}^{1} 1325 \cdot \frac{\sqrt{3}}{4} 1^{2} \cdot 9.8 \cdot(2-y) d y=\frac{1325 \cdot 9.8 \sqrt{3}}{4}\left[2 y-\frac{y^{2}}{2}\right]_{y=0}^{y=1}=\frac{1325 \cdot 9.8 \sqrt{3}}{4} \cdot \frac{3}{2} \text { Joules }
$$

