1. [12 points] Suppose that f is a twice differentiable function with continuous second derivative. (That is, both f and f' are differentiable, and f'' is continuous.) The following table gives some values of f and f'.

x	0	1	2	3	4	5	6	e^3
$f(x)$	7	5	-1	0	11	-3	2	9
$f'(x)$	3	-4	-2	4	-5	0	-1	2

In parts (a) through (c) below, calculate the exact numerical value of the integral. Write "NOT ENOUGH INFO" if there is not enough information to find the exact value. Be sure to show your work clearly. No partial credit will be given for estimates.

a. [4 points] $\displaystyle\int_1^{e^3} \frac{f'(\ln x)}{x}\,dx$

Solution: The substitution $w = \ln(x)$ gives $dw = \frac{dx}{x}$ and

$$\int_1^{e^3} \frac{f'(\ln x)}{x}\,dx = \int_0^3 f'(w)\,dw = f(3) - f(0) = 0 - 7 = -7.$$

b. [4 points] $\displaystyle\int_0^4 x f''(x)\,dx$

Solution: Integration by parts with $u = x$ and $dv = f''(x)\,dx$ gives

$$\begin{aligned}\int_0^4 x f''(x)\,dx &= x f'(x)\Big|_0^4 - \int_0^4 f'(x)\,dx \\ &= \big(4 \cdot f'(4) - 0 \cdot f'(0)\big) - \big(f(4) - f(0)\big) \\ &= -20 - (11 - 7) = -24.\end{aligned}$$

c. [4 points] $\displaystyle\int_2^6 f'(x)\,[f(x)]^2\,dx$

Solution:
One Approach: substitution with $w = f(x)$ so $dw = f'(x)\,dx$

$$\int_2^6 f'(x)\,[f(x)]^2\,dx = \int_{f(2)}^{f(6)} w^2\,dw = \frac{w^3}{3}\Big|_{-1}^{2} = \frac{8}{3} - \frac{-1}{3} = 3.$$

Another Approach: integration by parts with $u = f(x)^2$ and $dv = f'(x)\,dx$

$$\int_2^6 f'(x)\,[f(x)]^2\,dx = [f(x)]^3\Big|_2^6 - 2\int_2^6 f'(x)[f(x)]^2\,dx.$$

Moving the last term to the left hand side and dividing both sides of the resulting equation by 3 gives

$$\int_2^6 f'(x)\,[f(x)]^2\,dx = [f(x)]^3\Big|_2^6 = \frac{8 - (-1)}{3} = 3.$$