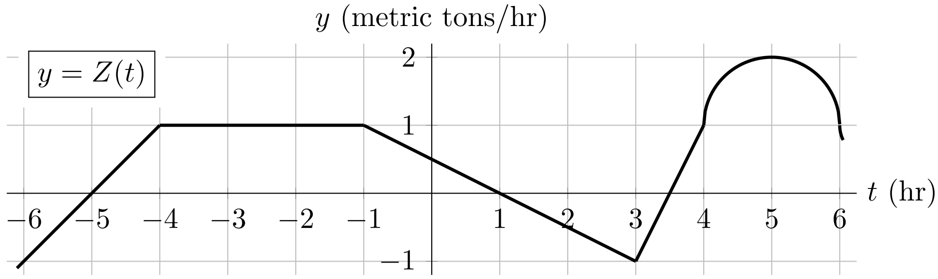
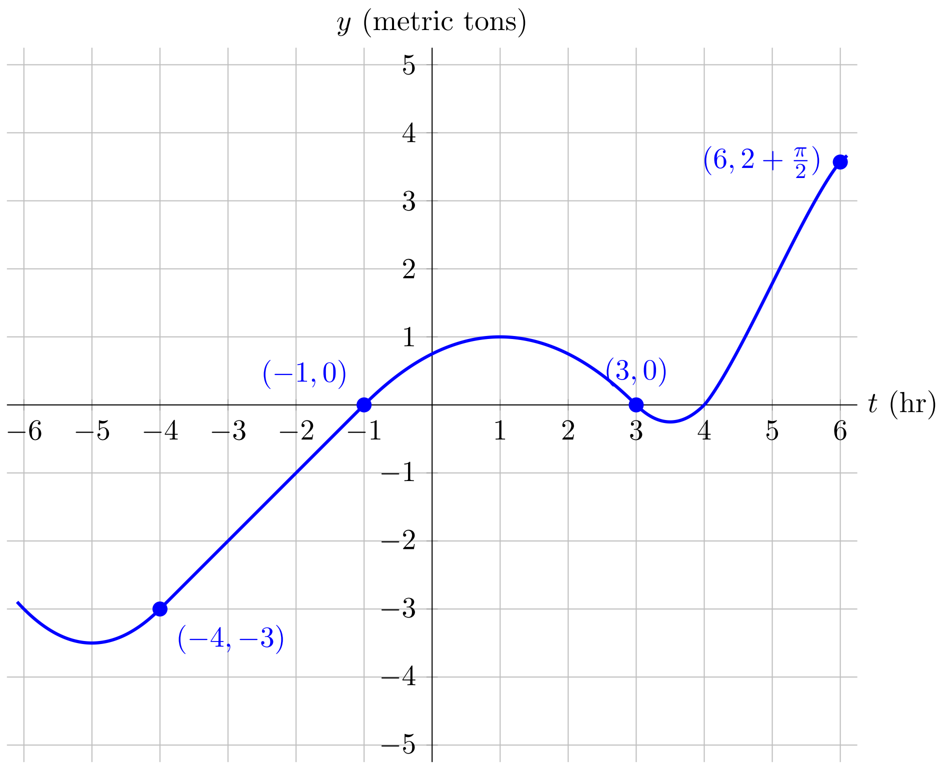
2. [13 points] Suppose $Z(t)$ is the rate of change, in metric tons per hour, of the biomass (i.e. total mass) of zooplankton in Loch Ness t hours after 8am on January 25, 2017.
Below is a portion of the graph of $Z(t)$. Note that this graph is linear on the intervals $[-6,-4]$, $[-4,-1]$, $[-1,3]$, and $[3,4]$. Also note that the portion of the graph for $4 \le t \le 6$ is the upper half of a circle centered at the point $(5,1)$.



Let $B(t)$ be the biomass, in metric tons, of zooplankton in Loch Ness t hours after 8am on January 25, 2017.

a. [10 points] Carefully sketch a graph of $y = B(t) - B(3)$ for $-6 \le t \le 6$ using the axes provided below. If there are features of this function that are difficult for you to draw, indicate these on your graph. Be sure that local extrema and concavity are clear.
Label the coordinates of the points on your graph at $t = -4, -1, 3, 6$.

Solution: Note that $B(t) - B(3) = \int_3^t Z(x)\,dx$ is the antiderivative of $Z(t)$ whose value is 0 at $t = 3$. Although it is difficult to tell here, the graph below is concave up for $4 < t < 5$ and concave down for $5 < t < 6$.



b. [3 points] Define $A(h)$ to be the average biomass (in metric tons) of zooplankton in Loch Ness during the first h hours after 8am on January 25, 2017. Write an expression for $A(h)$. (Your expression may involve integrals, the function Z, and/or the function B.)

Solution: $A(h)$ is the average value of the function $B(t)$ over the interval $0 \le t \le h$, so

$$A(h) = \frac{1}{h}\int_0^h B(t)\,dt.$$