5. [12 points] Several high-ranking Illuminati officials are relaxing while counting their money. There is so much money to count that the process takes many hours. The rate (in millions of dollars per hour) at which they count the money is given by the function $M(t)$, where t is the number of hours since they began counting. Several values of this function are given in the table below.

t (hours)	0	2	4	6	10	14
$M(t)$ (million \$/hour)	5	6	11	12	10	3

Note: The function $M(t)$ is continuous. Between each of the values of t given in the table, the function $M(t)$ is always increasing or always decreasing.

a. [2 points] Write, but do **not** evaluate, a definite integral that gives the total amount of money, in millions of dollars, the officials counted from the time they started until the time when they were counting the money the fastest.

Solution: $\int_0^6 M(t)\,dt$

b. [3 points] Write out the terms of a left Riemann sum with 3 equal subdivisions to estimate the integral from (a). Does this sum give an overestimate or an underestimate of the integral?

Solution: This left Riemann sum is $2 \cdot 5 + 2 \cdot 6 + 2 \cdot 11 = 44$.
Since $M(t)$ is increasing on the interavel $[0, 6]$, this is an underestimate of the integral from (a).

c. [4 points] Based on the data provided, write a sum that gives the best possible **overestimate** for the total amount of money, in millions of dollars, counted during the first 14 hours of counting.

Solution: The total amount of money, in millions of dollars, counted during the first 14 hours of counting is equal to $\int_0^{14} M(t)\,dt$.
The best possible overestimate of this integral is given by the Riemann sum that uses right endpoints on the intervals over which f is increasing and left endpoints on the intervals over which f is decreasing. This gives the estimate $2 \cdot (6 + 11 + 12) + 4 \cdot (12 + 10)$.

d. [3 points] What is the difference, in millions of dollars, between the best possible overestimate and the best possible underestimate for the total amount of money counted during the first 14 hours of counting?

Solution: We calculate the difference between the left and right hand sums on the intervals over which f is increasing and decreasing respectively and add these together, giving a difference of $2\,|12 - 5| + 4\,|3 - 12| = 50$.