6. [10 points] The Rodin Coil is a fantastic device that (supposedly) creates unlimited free energy. The rate at which it creates this energy is a function of the volume of the coil.

a. [5 points] Suppose a prototype of the Rodin Coil is the solid whose base is the circle $x^2+y^2=2$ (where x and y are measured in meters), and whose cross sections perpendicular to the x–axis are squares. Write, but do **not** compute, an expression involving one or more integrals which gives the volume, in cubic meters, of this prototype.

Solution: We find that the sidelength of the square at x–coordinate x is $2 \cdot \sqrt{2-x^2}$. So the volume of a slice of thickness Δx at that point is approximately $\left(2 \cdot \sqrt{2-x^2}\right)^2 \cdot \Delta x$. Integrating from the left end of the circle to the right end we find a total volume (in cubic meters) of

$$\int_{-\sqrt{2}}^{\sqrt{2}} \left(2\sqrt{2-x^2}\right)^2 \, dx.$$

b. [5 points] One of Rodin's students was able to come up with an even more efficient free energy machine. Suppose the student's prototype was made by taking the same circle $x^2+y^2=2$ and rotating it around the vertical line $x=3$. Write, but do **not** compute, an expression involving one or more integrals which gives the volume, in cubic meters, of this prototype.

Solution:
One Solution: Using cylindrical shells perpendicular to the x–axis, we find that the volume is equal to

$$\int_{-\sqrt{2}}^{\sqrt{2}} 2\pi(3-x)\left(2\sqrt{2-x^2}\right) \, dx.$$

Another Solution: Using slices of the solid perpendicular to the y–axis ("washers"), we find that the volume is equal to

$$\int_{-\sqrt{2}}^{\sqrt{2}} \pi \left[\left(3+\sqrt{2-y^2}\right)^2 - \left(3-\sqrt{2-y^2}\right)^2\right] dy.$$