7. [10 points] Consider the function F defined for all x by the formula

$$F(x) = \int_7^{x^2} e^{-t^2}\, dt.$$

a. [1 point] Find a number $a \geq 0$ so that $F(a) = 0$.

Solution: $a = \sqrt{7}$.

b. [4 points]

(i) Calculate $F'(x)$. Your answer should not contain any integrals.

Solution: Applying the Second Fundamental Theorem of Calculus and the Chain Rule, we find

$$F'(x) = e^{-(x^2)^2} \cdot 2x = 2xe^{-x^4}.$$

(ii) Is $F(x)$ increasing on the entire interval $[1, 8]$? Why or why not?

Solution: $F'(x) > 0$ if $x > 1$ (in fact, if $x > 0$). Thus $F(x)$ is increasing on this interval. Alternatively, $F(x)$ is the integral of a positive function, and the interval expands as x increases.

c. [3 points] Write out each term of a MID(3) estimate of $F(5)$.
(You do **not** need to find or approximate the numerical value of your answer.)

Solution: $F(5) = \int_7^{25} e^{-x^2}\, dx$, and therefore

$$MID(3) = 6 \cdot (e^{-10^2} + e^{-16^2} + e^{-22^2}) = 6(e^{-100} + e^{-256} + e^{-484}).$$

d. [2 points] Is your answer to part (c) an overestimate or underestimate of $F(5)$?
Briefly explain your reasoning.

Solution: Since e^{-x^2} is concave up on the interval $[7, 25]$, MID(3) is an underestimate of $F(5)$.