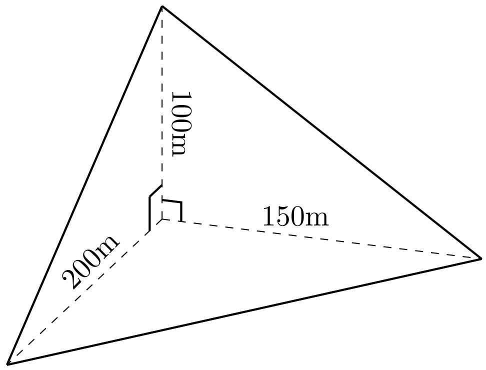
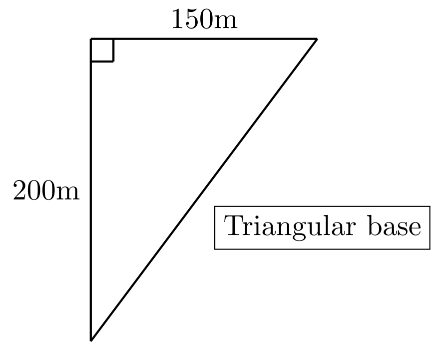
9. [11 points] Advanced beings from another planet recently realized they left a stockpile of nanotechnology here on Earth. These tiny devices are stacked in the shape of a pyramid with a triangular base that is flat on the ground. Its base is a right triangle with perpendicular sides of length 150m and 200m. Two of the other three sides are also right triangles, and all three right angles meet at one corner at the base of the pile. The fourth side is a triangle whose sides are the hypotenuses of the other three triangles. (See diagrams below.)





The density of the contents of the pile at a height of h meters above the ground is given by

$$\delta(h) = \frac{2}{\sqrt{1+h^2}} \text{ kg/m}^3.$$

For this problem, you may assume the acceleration due to gravity is $g = 9.8$ m/s^2.

a. [4 points] Write an expression in terms of h that approximates the volume (in cubic meters) of a horizontal slice of thickness Δh of the contents of the pile at a distance h meters above the ground.

Solution: The cross section of the pile h meters above the ground is a right triangle with legs of length $200 - 2h$ and $150 - 1.5h$ meters. (One can use e.g. linearity or similar triangles to find these lengths.) Therefore, the volume (in cubic meters) of the horizontal slice is approximately $\frac{1}{2}(200-2h)(150-1.5h)\Delta h$.

b. [3 points] Write, but do **not** to evaluate, an integral that gives the total mass of the pile of nanotechnology. Include units.

Solution: We mutiply the expression in (a) by the density to get an approximation for the mass of the slice and integrate that expression from 0 to 100 while replacing Δh by dh to find a total mass of

$$\int_0^{100} \frac{1}{2}(200-2h)(150-1.5h)\frac{2}{\sqrt{1+h^2}}\,dh \quad \text{kilograms.}$$

c. [4 points] The beings must return to Earth and collect the nanotech that they left behind. Suppose that the spaceship hovers 150 meters above the ground (directly above the pile) while recovering the nanotechnology. Write, but do **not** evaluate, an integral which gives the total work that must be done in order to lift all of the nanotech from the pile into the ship. Include units.

Solution: We multiply the expression in (a) by the density, acceleration due to gravity, and the distance required to lift it to a height of 150 m to estimate the work done to lift that slice up to the spaceship. Then we integrate this expression from 0 to 100 and replace Δh by dh to find that the total work done is

$$\int_0^{100} g\cdot(150-h)\cdot\frac{1}{2}(200-2h)(150-1.5h)\frac{2}{\sqrt{1+h^2}}\,dh \quad \text{Joules.}$$