

10. [8 points] For each of the questions below, circle all of the available correct answers. Circle NONE OF THESE if none of the available choices are correct. You must circle at least one choice to receive any credit. No credit will be awarded for unclear markings. No justification is necessary.

a. [3 points] Which of the following integrals are equal to $\lim_{n \rightarrow \infty} \sum_{k=0}^{n-1} \left(3 + \frac{2k}{n}\right)^4 \cdot \frac{2}{n}$?

i. $\int (1 + kx)^4 x \, dx$

iv. $\int_2^3 x^4 \, dx$

ii. $\int_3^5 x^4 \, dx$

v. $\int_0^2 (3 + x)^4 \, dx$

iii. $\int_0^{n-1} \left(3 + \frac{2x}{n}\right)^4 \cdot \frac{2}{n} \, dx$

vi. NONE OF THESE

b. [3 points] Which of the following expressions give the volume of the solid made by rotating around the y -axis the region bounded by $y = \sin(x)$, $y = 0$, and $x = \frac{\pi}{2}$?

i. $\int_0^{\pi/2} \pi \left(\frac{\pi}{2} - \sin(x)\right)^2 \, dx$

v. $\int_0^1 \pi \left(\frac{\pi}{2} - \arcsin(y)\right)^2 \, dy$

ii. $\int_0^{\pi/2} \pi \left(\left(\frac{\pi}{2}\right)^2 - \sin^2(x)\right) \, dx$

vi. $\int_0^1 \pi \left(\left(\frac{\pi}{2}\right)^2 - (\arcsin(y))^2\right) \, dy$

iii. $\int_0^{\pi/2} 2\pi x \sin(x) \, dx$

vii. $\int_0^1 2\pi y \arcsin(y) \, dy$

iv. $\int_0^{\pi/2} \pi \sin^2(x) \, dx$

viii. NONE OF THESE

c. [2 points] Let $f(x)$ be a function that is increasing on $(-3, 3)$, concave up on $(0, 3)$, and has a point of inflection at $x = 0$. Consider the approximations for $\int_{-2}^2 f(x) \, dx$ given by LEFT(n) and TRAP(n). Which of the following statements **must** be true?

i. $\text{TRAP}(n) < \int_{-2}^2 f(x) \, dx$

iv. $\text{LEFT}(n) < \int_{-2}^2 f(x) \, dx$

ii. $\text{TRAP}(n) > \int_{-2}^2 f(x) \, dx$

v. $\text{LEFT}(n) > \int_{-2}^2 f(x) \, dx$

iii. TRAP(n) is neither an overestimate nor an underestimate for $\int_{-2}^2 f(x) \, dx$.

vi. $\text{LEFT}(n) = 0$

vii. NONE OF THESE