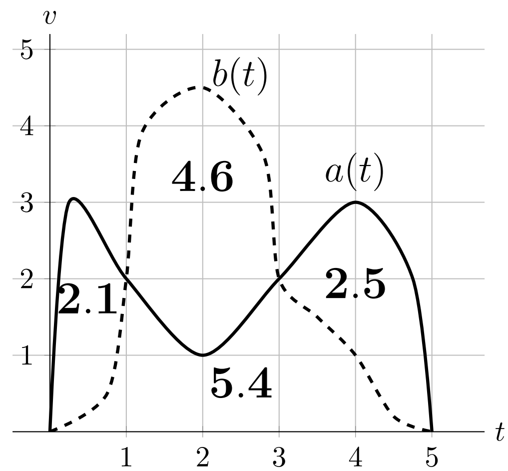
2. [9 points]

When Alejandra and Brontel were children they spent summer mornings chasing birds in flight. One memorable day they encountered an owl. The following graph shows the velocities $a(t)$ of Alejandra (solid) and $b(t)$ of Brontel (dashed), measured in meters per second, t seconds after the owl took off. The area of each region is given.



a. [1 point] How far (in meters) do Alejandra and Brontel chase the owl?

Solution: Summing the areas under either curve gives a total distance of 10 m.

b. [5 points] Suppose the owl ascends to a height of h meters according to $h(t) = \sqrt{t}$ where t is seconds since it went airborne. Let $L(h)$ be the number of meters Alejandra is ahead of Brontel when the owl is h meters above ground. Write an expression for $L(h)$ involving integrals and compute $L'(2)$.

Solution: The owl is h meters above the ground at time $t = h^2$. Thus,

$$L(h) = \int_0^{h^2} a(t) - b(t)\, dt.$$

We compute $L'(h)$ using the second fundamental theorem of calculus.

$$L'(h) = 2ha(h^2) - 2hb(h^2).$$

So we have

$$L'(2) = 2 \cdot 2a(4) - 2 \cdot 2b(4) = 4 \cdot 3 - 4 \cdot 1 = 8,$$

where we get the values of $a(2)$ and $b(2)$ from the graph.

c. [3 points] The next bird to pass is a dove. This time Alejandra runs twice as fast and Brontel runs three times as fast as they did when chasing the owl. How much faster (in m/s) is Brontel than Alejandra on average in the first 5 seconds?

Solution: The integral that represents this average is

$$\frac{1}{5}\int_0^5 3b(t) - 2a(t)\, dt = \frac{3}{5}\int_0^5 b(t)\, dt - \frac{2}{5}\int_0^5 a(t)\, dt.$$

Each of these integrals is equal to 10 as we see from the graph. Hence

$$\frac{1}{5}\int_0^5 3b(t) - 2a(t)\, dt = \frac{10}{5} = 2.$$

Answer: **2 m/s**