3. [10 points]

Debra McQueath hooked you up with an interview at Print.juice. Being a legitimate tech start-up, the Print.juice interview consists of answering technical questions on the spot. Debra gave you the following questions for practice.

The region $J$ is a common Print.juice shape. It is bounded by $x = 1$, $y = 1$, and $y = e^x$.

a. [3 points] First, consider the solid with base $J$ and square cross sections perpendicular to the $x$-axis. If the density of the solid is a function of the $x$-coordinate $a(x)$ g/cm$^3$, write an integral that represents the total mass of the solid in grams.

**Solution:** The height of a cross-section is $e^x - 1$, thus the total mass is

$$\int_0^1 a(x)(e^x - 1)^2 \, dx.$$ 

For b. and c., consider the solid made by rotating $J$ around the line $x = 2$.

b. [3 points] If the density of the solid is a function of the $y$-coordinate $b(y)$ g/cm$^3$, write an integral that represents the total mass of the solid in grams.

**Solution:** Using the washer method we compute the total mass to be

$$\int_1^e b(y)\pi((2 - \ln(y))^2 - 1^2) \, dy.$$ 

c. [4 points] If the density of the solid is a function of the distance $r$ cm from the axis of rotation $c(r)$ g/cm$^3$, write an integral that represents the total mass of the solid in grams.

**Solution:** Using the shell method we can either compute the mass in terms of $x$ or $r$. In terms of $r$ we get

$$\int_1^2 c(r)2\pi r(e^2-r-1) \, dr,$$

and in terms of $x$ we get

$$\int_0^1 c(2-x)2\pi(2-x)(e^x-1) \, dx.$$