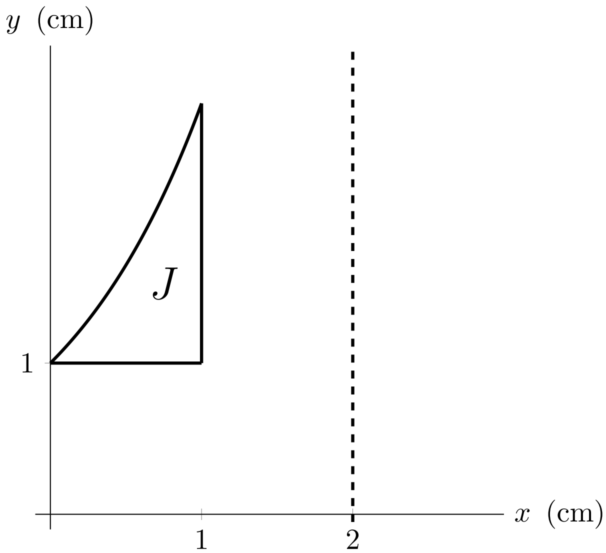
3. [10 points]

Debra McQueath hooked you up with an interview at `Print.juice`. Being a legitimate tech start-up, the `Print.juice` interview consists of answering technical questions on the spot. Debra gave you the following questions for practice.

The region J is a common `Print.juice` shape. It is bounded by $x = 1$, $y = 1$, and $y = e^x$.



a. [3 points] First, consider the solid with base J and square cross sections perpendicular to the x-axis. If the density of the solid is a function of the x-coordinate $a(x)$ g/cm^3, write an integral that represents the total mass of the solid in grams.

Solution: The height of a cross-section is $e^x - 1$, thus the total mass is

$$\int_0^1 a(x)(e^x - 1)^2\, dx.$$

For b. and c., consider the solid made by rotating J around the line $x = 2$.

b. [3 points] If the density of the solid is a function of the y-coordinate $b(y)$ g/cm^3, write an integral that represents the total mass of the solid in grams.

Solution: Using the washer method we compute the total mass to be

$$\int_1^e b(y)\pi((2 - \ln(y))^2 - 1^2)\, dy.$$

c. [4 points] If the density of the solid is a function of the distance r cm from the axis of rotation $c(r)$ g/cm^3, write an integral that represents the total mass of the solid in grams.

Solution: Using the shell method we can either compute the mass in terms of x or r. In terms of r we get

$$\int_1^2 c(r)2\pi r(e^{2-r} - 1)\, dr,$$

and in terms of x we get

$$\int_0^1 c(2 - x)2\pi(2 - x)(e^x - 1)\, dx.$$