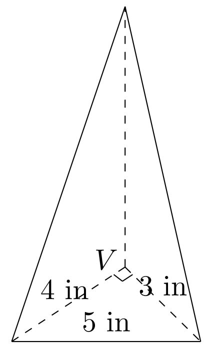
7. [10 points]

Ms. Parth made a pyramid for her niece and nephew. The pyramid is 10 inches tall and the base has the shape of a right triangle. When the pyramid is sitting on the table it looks like the figure to the right. The three angles at vertex V are right angles. (Dashed lines are not visible from this point of view. The figure may not be drawn to scale.)



a. [6 points] Write an integral that represents the total volume of the pyramid in cubic inches and evaluate it.

Solution: Let h represent inches from the top of the pyramid. A cross-section h inches from the top of the pyramid and parallel to the table is a right triangle similar to the base. Using similar triangles we find that the cross-section has side lengths $\frac{3}{10}h$ and $\frac{4}{10}h$. Hence the integral representing its volume is

$$\begin{aligned}\text{Volume} &= \int_0^{10} \frac{1}{2}\left(\frac{3}{10}\right)\left(\frac{4}{10}\right) h^2\, dh \\ &= \frac{6}{10^2}\int_0^{10} h^2\, dh.\end{aligned}$$

Then the fundamental theorem of calculus gives

$$\text{Volume} = \frac{6}{10^2}\cdot \frac{h^3}{3}\bigg|_0^{10} = 20\,\text{in}^3.$$

If y represents inches above the table, then the integral will be

$$\text{Volume} = \int_0^{10} \frac{1}{2}\left(\frac{3}{10}\right)\left(\frac{4}{10}\right)(10-y)^2\, dy.$$

b. [4 points] The children fail to share the pyramid, so Ms. Parth decides to cut it parallel to the table into two pieces of equal volume. How many inches H from the **top** of the pyramid should Ms. Parth cut? Round your answer to the nearest tenth of an inch.

Solution: Since the volume of the pyramid is 20 in^3, we want to find H satisfying

$$10 = \int_0^{H} \frac{1}{2}\left(\frac{3}{10}\right)\left(\frac{4}{10}\right) h^2\, dh.$$

Using the fundamental theorem of calculus this becomes

$$10 = \frac{2}{100}H^3 \longrightarrow H = \frac{10}{\sqrt[3]{2}} \approx 7.937.$$