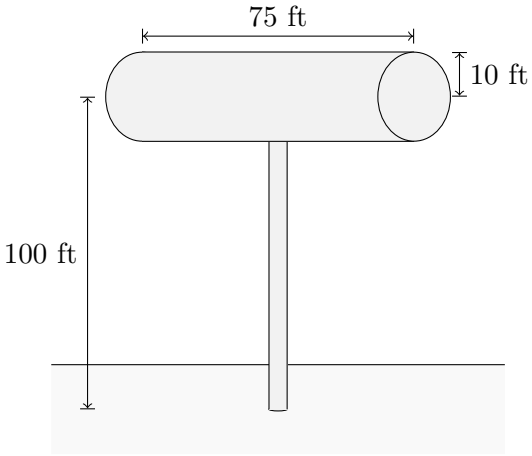
9. [9 points]

De'von Baptiste is a shrewd industrialist. When energy costs are low, De'von pumps purified muck (which he gets *for free* from the city) into very tall tanks. In this way he stores cheap potential energy. Someday, when energy prices soar, Mr. Batiste will convert it all back into useful kinetic energy at a great profit.

His tanks are cylinders 75 ft long with radius 10 ft. The center of a tank is 100 ft above the ground. Purified muck has a density of 800 pounds/ft^3.



a. [3 points] What is the area, in square feet, of a cross-section parallel to the ground taken y feet above the **center** of the tank?

Solution: The cross-sections are rectangles with a length of 75 ft and a width $w(y)$ which depends on y. Using the Pythagorean Theorem we find that

$$10^2 = y^2 + (w(y)/2)^2 \longrightarrow w(y) = 2\sqrt{100 - y^2} = \sqrt{400 - 4y^2}.$$

Hence the area of a cross-section is

$$\text{Area of cross-section} = 75w(y) = 75 \cdot 2\sqrt{100 - y^2} = 150\sqrt{100 - y^2}.$$

Answer: $150\sqrt{100 - y^2}$

b. [6 points] Write an integral which represents the total work (in foot-pounds) required to fill one of De'von Batiste's tanks with purified muck. **Do not evaluate this integral.**

Solution: If we consider the tank after its filled, we can compute the work required to get each slice of muck y feet above the center of the tank from the ground to its height at $100 + y$ ft above the ground. If $A(y) = 300\sqrt{100 - y^2}$ is the area of a cross-section y feet above the center of the tank, then the total work is

$$\begin{aligned}\text{Total work} &= \int_{-10}^{10} (\text{density})(\text{distance})(\text{slice volume}) \\ &= \int_{-10}^{10} 800(100 + y)A(y)\, dy \\ &= \int_{-10}^{10} 800(100 + y)150\sqrt{100 - y^2}\, dy\end{aligned}$$