3. [11 points] The parts of this problem are not related.

a. [6 points] Suppose $f(x)$ is a positive function, defined for all real numbers x, with continuous first derivative. For each part below, circle "True" if the statement is **always** true and circle "False" otherwise. No justification is necessary.

$\int_0^3 xf(x^2)\,dx = \frac{1}{2}\int_0^3 f(u)\,du$	TRUE	FALSE
$\int_0^3 xf(x^2)\,dx = \int_0^3 sf(s^2)\,ds$	TRUE	FALSE
$\int xf(x^2)\,dx = x \cdot \int f(x^2)\,dx$	TRUE	FALSE
$\int xf(x^2)\,dx = x \cdot \int f(x^2)\,dx + f(x^2) \cdot \int x\,dx$	TRUE	FALSE
$\int xf(x^2)\,dx = \int x\,dx \cdot \int f(x^2)\,dx$	TRUE	FALSE
$\int xf(x^2)\,dx = \frac{x^2}{2} f'(x^2) - \int x^3 f'(x^2)\,dx$	TRUE	FALSE

b. [2 points] Suppose $G(x)$ and $H(x)$ are continuous antiderivatives of an even function $g(x)$ and $G(1) > H(1)$. Which of the following must be true?

i. $G(-1)$ is definitely greater than $H(-1)$.

ii. $G(-1)$ is definitely not greater than $H(-1)$.

iii. None of these.

c. [3 points] A region bounded entirely by the graph of the function $y = \arctan(x)$, the y-axis, and the line $y = \frac{\pi}{4}$ is rotated around the x-axis. Which of the following integrals represents the volume of the resulting solid? Choose the <u>one</u> best answer.

i. $\pi \int_0^1 \left(\frac{\pi}{4} - \arctan(x)\right)^2 dx$

ii. $\pi \int_0^1 \left(\frac{\pi}{4}\right)^2 - (\arctan(x))^2\,dx$

iii. $\pi \int_0^{\pi/4} 1 - (\arctan(x))^2\,dx$

iv. $\pi \int_0^{\pi/4} (\tan(y))^2 - 1\,dy$

v. $\pi \int_0^{\pi/4} (\tan(y) - 1)^2\,dy$

vi. $\pi \int_0^1 \left(\tan(y) - \frac{\pi}{4}\right)^2 dy$

vii. $\pi \int_0^1 (\tan(y))^2 - \left(\frac{\pi}{4}\right)^2 dy$

viii. NONE OF THESE