5. [9 points] Before calculators existed, it was a difficult task for scientists and engineers to compute values of special functions like logarithm. In this problem, we will use simple arithmetic operations to approximate the value of $\ln 2$.
Note that for $x > 0$,

$$\ln x = \int_1^x \frac{1}{t}\,dt.$$

a. [3 points] Approximate the integral $\displaystyle\int_1^2 \frac{1}{t}\,dt$ using LEFT(4). Write out each term in your sum.

Answer: ____________________________

b. [3 points] Which of the following are equal to the LEFT(n) approximations for $\displaystyle\int_1^2 \frac{1}{t}\,dt$? Circle the <u>one</u> best answer.

i. $\displaystyle\frac{1}{n}\sum_{i=0}^{n-1}\frac{1}{i}$

ii. $\displaystyle\frac{1}{n}\sum_{i=0}^{n}\frac{1}{1+i/n}$

iii. $\displaystyle\frac{1}{n}\sum_{i=0}^{n-1}\frac{1}{1+i/n}$

iv. $\displaystyle\frac{1}{n}\sum_{i=1}^{n}\frac{1}{i}$

v. $\displaystyle\frac{1}{n}\sum_{i=n}^{n}\frac{1}{1+i/n}$

vi. $\displaystyle\frac{1}{n}\sum_{i=0}^{n}\frac{1}{1+1/n}$

c. [3 points] How many subintervals would be needed so that a scientist who lived before calculators were invented would be certain that the resulting left-hand Riemann sum approximates ln(2) to within 0.01? Justify your answer.

Answer: ____________________________