6. [12 points] Ryan Rabbitt is making a smoothie with his new electric drink mixer. Mathematically, the container of the mixer has a shape that can be modeled as the surface obtained by rotating the region in the first quadrant bounded by the curves $y=27$ and $y=x^{3 / 2}$ about the $y$-axis, where all lengths are measured in centimeters.
a. [7 points] Write, but do not evaluate, two integrals representing the total volume, in $\mathrm{cm}^{3}$, the mixer can hold: one with respect to $x$, and one with respect to $y$.

Answer (with respect to $x$ ): $\qquad$

Answer (with respect to $y$ ):
b. [5 points] Ryan adds 1600 cubic centimeters of liquid to his mixer. The container spins around the $y$-axis at a very high speed, causing the liquid to move away from the center of the container. The result is the solid made by rotating the shaded region around the $y$-axis in the diagram below. Note that this means that there is an empty space inside the liquid that has the shape of a cylinder.


Let $r$ be the radius of this cylinder of empty space. Set up an equation involving one or more integrals that you would use to solve to find the value of $r$. Do not solve for $r$.

## Answer:

