1. [11 points] Let $f(x)$ be a differentiable function with continuous derivative, and $F(x) = \int_2^x f(t)\,dt$. Some values of the functions $f(x)$ and $F(x)$ are shown below:

x	-1	0	1	2	3
$f(x)$	7	4	0.25	9	8
$F(x)$	8	9	0.5	0	1

Compute the exact numerical values of the following integrals. If it is not possible to do so based on the information provided, write "NOT POSSIBLE" and clearly indicate why it is not possible. Show your work.

a. [3 points] $\int_0^1 xf'(x)\,dx$

Solution: By parts.

$$\begin{aligned}\int_0^1 xf'(x)\,dx &= xf(x)\Big|_0^1 - \int_0^1 f(x)\,dx \\ &= xf(x)\Big|_0^1 - F(x)\Big|_0^1 \\ &= f(1) - F(1) + F(0) = 0.25 - 0.5 + 9 = 8.75\end{aligned}$$

Answer: $8.75 = 35/4$

b. [3 points] $\int_{-1}^0 \sqrt[3]{x} f'(x^{4/3})\,dx$

Solution: Use substitution with $u = x^{4/3}$.

$$\int_{-1}^0 \sqrt[3]{x} f'(x^{4/3})\,dx = \frac{3}{4}\int_1^0 f'(u)\,du = \frac{3}{4}\,f(u)|_1^0 = \frac{3}{4}(4 - 0.25) = \frac{45}{16} = 2.8125$$

Answer: $45/16 = 2.8125$

c. [5 points] $\int_1^2 \frac{f(x)}{(F(x))^2 - 1}\,dx$

Solution: Substitute $u = F(x)$ and then use partial fractions.

$$\begin{aligned}\int_1^2 \frac{f(x)}{(F(x))^2-1}\,dx = \int_{F(1)}^{F(2)} \frac{1}{u^2-1}\,du &= \int_{0.5}^0 \frac{1/2}{u-1} - \frac{1/2}{u+1}\,du \\ &= \frac{1}{2}\ln|u-1|\Big|_{0.5}^0 - \frac{1}{2}\ln|u+1|\Big|_{0.5}^0 \\ &= \frac{1}{2}(\ln 1 - \ln 0.5) - \frac{1}{2}(\ln 1 - \ln 1.5) \\ &= \frac{1}{2}(\ln 1.5 - \ln 0.5) = \frac{1}{2}\ln 3 = \ln\sqrt{3}\end{aligned}$$

Answer: $(\ln 1.5 - \ln 0.5)/2 = \ln 3/2 = \ln\sqrt{3}$