

3. [11 points] The parts of this problem are not related.

- a. [6 points] Suppose  $f(x)$  is a positive function, defined for all real numbers  $x$ , with continuous first derivative. For each part below, circle “True” if the statement is **always** true and circle “False” otherwise. No justification is necessary.

$$\int_0^3 x f(x^2) dx = \frac{1}{2} \int_0^3 f(u) du \quad \text{TRUE} \quad \boxed{\text{FALSE}}$$

$$\int_0^3 x f(x^2) dx = \int_0^3 s f(s^2) ds \quad \boxed{\text{TRUE}} \quad \text{FALSE}$$

$$\int x f(x^2) dx = x \cdot \int f(x^2) dx \quad \text{TRUE} \quad \boxed{\text{FALSE}}$$

$$\int x f(x^2) dx = x \cdot \int f(x^2) dx + f(x^2) \cdot \int x dx \quad \text{TRUE} \quad \boxed{\text{FALSE}}$$

$$\int x f(x^2) dx = \int x dx \cdot \int f(x^2) dx \quad \text{TRUE} \quad \boxed{\text{FALSE}}$$

$$\int x f(x^2) dx = \frac{x^2}{2} f'(x^2) - \int x^3 f'(x^2) dx \quad \text{TRUE} \quad \boxed{\text{FALSE}}$$

- b. [2 points] Suppose  $G(x)$  and  $H(x)$  are continuous antiderivatives of an even function  $g(x)$  and  $G(1) > H(1)$ . Which of the following must be true?

- $G(-1)$  is definitely greater than  $H(-1)$ .
- $G(-1)$  is definitely not greater than  $H(-1)$ .
- None of these.

- c. [3 points] A region bounded entirely by the graph of the function  $y = \arctan(x)$ , the  $y$ -axis, and the line  $y = \frac{\pi}{4}$  is rotated around the  $x$ -axis. Which of the following integrals represents the volume of the resulting solid? Choose the one best answer.

$$\text{i. } \pi \int_0^1 \left( \frac{\pi}{4} - \arctan(x) \right)^2 dx$$

$$\text{v. } \pi \int_0^{\pi/4} (\tan(y) - 1)^2 dy$$

$$\text{ii. } \pi \int_0^1 \left( \frac{\pi}{4} \right)^2 - (\arctan(x))^2 dx$$

$$\text{vi. } \pi \int_0^1 \left( \tan(y) - \frac{\pi}{4} \right)^2 dy$$

$$\text{iii. } \pi \int_0^{\pi/4} 1 - (\arctan(x))^2 dx$$

$$\text{vii. } \pi \int_0^1 (\tan(y))^2 - \left( \frac{\pi}{4} \right)^2 dy$$

$$\text{iv. } \pi \int_0^{\pi/4} (\tan(y))^2 - 1 dy$$

viii. NONE OF THESE