**4.** [11 points] A polar vortex arrives in a college town at midnight, causing the temperature to drop. Consider the following:

- Let $t$ be the time, in hours, after the polar vortex arrives.
- Let $r(t)$ be the rate, in degrees Fahrenheit per hour, at which the temperature is changing at time $t$.
- At first the temperature drops quickly, but as time passes, it drops less quickly.
- When the polar vortex first arrives, the temperature is 22° F.

Some values of $r(t)$ are given in the table below.

| $t$ | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|
| $r(t)$ | $-8$ | $-5$ | $-3$ | $-2$ | $-1$ |

**a.** [2 points] Which of the following expressions must be the average rate of change of the temperature between midnight and 6 am? Circle all correct answers.

$\boxed{\frac{1}{6}\int_0^6 r(t)\,dt}$ $\quad \frac{1}{6}\int_0^6 r'(t)\,dt$ $\quad \frac{\int_0^6 r(t)\,dt - 22}{6}$

$\frac{r(0)+r(2)+r(4)+r(6)}{4}$ $\quad \frac{r(6)-r(0)}{6}$ $\quad$ NONE OF THESE

**b.** [3 points] Write an expression involving a definite integral that represents the temperature, in degrees Fahrenheit, at 8 am.

**Answer:** $22 + \int_0^8 r(t)\,dt$

**c.** [6 points] At 8:00 am, Alexis is walking to class, and says, "I can't believe they didn't cancel classes! It must be 20 below out here!" Assuming Alexis means that the temperature is less than or equal to $-20°$ F, is Alexis correct? Circle your answer, and show work or explain your reasoning below.

ALEXIS IS CORRECT $\quad$ $\boxed{\text{ALEXIS IS INCORRECT}}$ $\quad$ CANNOT BE DETERMINED

**Justification:**

*Solution:* We are told that $r(t)$ is increasing ("At first the temperature drops quickly, but as time passes, it drops less quickly"), so LEFT will be an underestimate, which is what we need in this case. (We are not told the concavity of $r(t)$, but graphing the points we have make it seem likely that the function will be concave down, which would make TRAP an underestimate as well.)
Using LEFT(4) to approximate $\int_0^8 r(t)\,dt + 22$ gives us
LEFT(4) $= 2(-8 + -5 + -3 + -2) = -36$, so $22 + -36 = -14$

Since LEFT(4) is an underestimate, the actual temperature cannot be less than $-14$, and therefore Alexis is incorrect.