5. [9 points] Before calculators existed, it was a difficult task for scientists and engineers to compute values of special functions like logarithm. In this problem, we will use simple arithmetic operations to approximate the value of \( \ln 2 \).

Note that for \( x > 0 \),

\[
\ln x = \int_1^x \frac{1}{t} \, dt.
\]

a. [3 points] Approximate the integral \( \int_1^2 \frac{1}{t} \, dt \) using \( \text{LEFT}(4) \). Write out each term in your sum.

\[
\text{Solution:}
\]

\[
\text{LEFT}(4) = \frac{2 - 1}{4} \left( \frac{1}{1} + \frac{1}{1 + 1/4} + \frac{1}{1 + 2/4} + \frac{1}{1 + 3/4} \right)
\]

\[
= \frac{1}{4} \left(1 + \frac{4}{5} + \frac{4}{6} + \frac{4}{7} \right)
\]

\[
= \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}.
\]

Answer: \( \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} \)

b. [3 points] Which of the following are equal to the \( \text{LEFT}(n) \) approximations for \( \int_1^2 \frac{1}{t} \, dt \)?

Circle the one best answer.

i. \( \frac{1}{n} \sum_{i=0}^{n-1} \frac{1}{i} \)

ii. \( \frac{1}{n} \sum_{i=0}^{n} \frac{1}{1 + i/n} \)

iii. \( \frac{1}{n} \sum_{i=0}^{n-1} \frac{1}{1 + i/n} \)

iv. \( \frac{1}{n} \sum_{i=1}^{n} \frac{1}{i} \)

v. \( \frac{1}{n} \sum_{i=n}^{n} \frac{1}{1 + i/n} \)

vi. \( \frac{1}{n} \sum_{i=0}^{n} \frac{1}{1 + 1/n} \)

c. [3 points] How many subintervals would be needed so that a scientist who lived before calculators were invented would be certain that the resulting left-hand Riemann sum approximates \( \ln(2) \) to within 0.01? Justify your answer.

\[
\text{Solution:}
\]

\[
|\text{LEFT}(n) - \text{RIGHT}(n)| \leq \left| \frac{1}{1} - \frac{1}{2} \right| \cdot \frac{2 - 1}{n} = \frac{1}{2} \cdot \frac{1}{n} = \frac{1}{2n}
\]

Since \( 1/2n \leq 0.01 \) if and only if \( n \geq 50 \), at least 50 subintervals would be needed.

Answer: \( (\text{at least}) \ 50 \)