5. [9 points] Before calculators existed, it was a difficult task for scientists and engineers to compute values of special functions like logarithm. In this problem, we will use simple arithmetic operations to approximate the value of $\ln 2$.
Note that for $x > 0$,

$$\ln x = \int_1^x \frac{1}{t}\,dt.$$

a. [3 points] Approximate the integral $\int_1^2 \frac{1}{t}\,dt$ using LEFT(4). Write out each term in your sum.

Solution:

$$\begin{aligned}\text{LEFT}(4) &= \frac{2-1}{4}\left(\frac{1}{1} + \frac{1}{1+1/4} + \frac{1}{1+2/4} + \frac{1}{1+3/4}\right)\\ &= \frac{1}{4}\left(1 + \frac{4}{5} + \frac{4}{6} + \frac{4}{7}\right)\\ &= \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}.\end{aligned}$$

Answer: $\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$

b. [3 points] Which of the following are equal to the LEFT(n) approximations for $\int_1^2 \frac{1}{t}\,dt$? Circle the one best answer.

i. $\frac{1}{n}\sum_{i=0}^{n-1} \frac{1}{i}$

ii. $\frac{1}{n}\sum_{i=0}^{n} \frac{1}{1+i/n}$

iii. $\boxed{\frac{1}{n}\sum_{i=0}^{n-1} \frac{1}{1+i/n}}$

iv. $\frac{1}{n}\sum_{i=1}^{n} \frac{1}{i}$

v. $\frac{1}{n}\sum_{i=n}^{n} \frac{1}{1+i/n}$

vi. $\frac{1}{n}\sum_{i=0}^{n} \frac{1}{1+1/n}$

c. [3 points] How many subintervals would be needed so that a scientist who lived before calculators were invented would be certain that the resulting left-hand Riemann sum approximates $\ln(2)$ to within 0.01? Justify your answer.

Solution:

$$|\text{LEFT}(n) - \text{RIGHT}(n)| \le \left|\frac{1}{1} - \frac{1}{2}\right| \cdot \frac{2-1}{n} = \frac{1}{2}\cdot\frac{1}{n} = \frac{1}{2n}$$

Since $1/2n \le 0.01$ if and only if $n \ge 50$, at least 50 subintervals would be needed.

Answer: **(at least)** 50