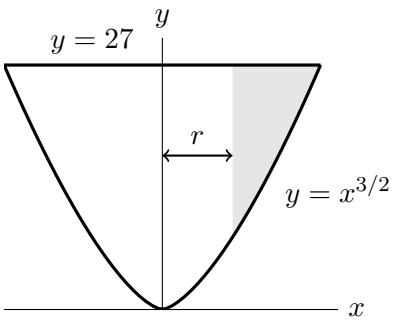
6. [12 points] Ryan Rabbitt is making a smoothie with his new electric drink mixer. Mathematically, the container of the mixer has a shape that can be modeled as the surface obtained by rotating the region in the first quadrant bounded by the curves $y = 27$ and $y = x^{3/2}$ about the y-axis, where all lengths are measured in centimeters.

a. [7 points] Write, but do not evaluate, two integrals representing the total volume, in cm^3, the mixer can hold: one with respect to x, and one with respect to y.

Answer (with respect to x): $\displaystyle\int_0^9 2\pi x \left(27 - x^{3/2}\right)\, dx$

Answer (with respect to y): $\displaystyle\int_0^{27} \pi \left(y^{2/3}\right)^2\, dy$

b. [5 points] Ryan adds 1600 cubic centimeters of liquid to his mixer. The container spins around the y-axis at a very high speed, causing the liquid to move away from the center of the container. The result is the solid made by rotating the shaded region around the y-axis in the diagram below. Note that this means that there is an empty space inside the liquid that has the shape of a cylinder.



Let r be the radius of this cylinder of empty space. Set up an equation involving one or more integrals that you would use to solve to find the value of r. **Do <u>not</u> solve for r.**

Solution:

$$\int_r^9 2\pi x \left(27 - x^{3/2}\right)\, dx = 1600,$$

or

$$\int_{r^{3/2}}^{27} \pi \left(y^{2/3}\right)^2\, dy - \pi r^2 (27 - r^{3/2}) = 1600.$$

(There are other equations that would also work.)

Answer: