6. [12 points] Ryan Rabbitt is making a smoothie with his new electric drink mixer. Mathematically, the container of the mixer has a shape that can be modeled as the surface obtained by rotating the region in the first quadrant bounded by the curves \( y = 27 \) and \( y = x^{3/2} \) about the \( y \)-axis, where all lengths are measured in centimeters. 

a. [7 points] Write, but do not evaluate, two integrals representing the total volume, in cm\(^3\), the mixer can hold: one with respect to \( x \), and one with respect to \( y \).

**Answer** (with respect to \( x \)): \( \int_{0}^{9} 2\pi x \left( 27 - x^{3/2} \right) \, dx \)

**Answer** (with respect to \( y \)): \( \int_{0}^{27} \pi \left( y^{2/3} \right)^2 \, dy \)

b. [5 points] Ryan adds 1600 cubic centimeters of liquid to his mixer. The container spins around the \( y \)-axis at a very high speed, causing the liquid to move away from the center of the container. The result is the solid made by rotating the shaded region around the \( y \)-axis in the diagram below. Note that this means that there is an empty space inside the liquid that has the shape of a cylinder.

Let \( r \) be the radius of this cylinder of empty space. Set up an equation involving one or more integrals that you would use to solve to find the value of \( r \). **Do not solve for** \( r \).

**Solution:**

\[
\int_{r}^{9} 2\pi x \left( 27 - x^{3/2} \right) \, dx = 1600,
\]

or

\[
\int_{r^{3/2}}^{27} \pi \left( y^{2/3} \right)^2 \, dy - \pi r^2 (27 - r^{3/2}) = 1600.
\]

(There are other equations that would also work.)

**Answer:**