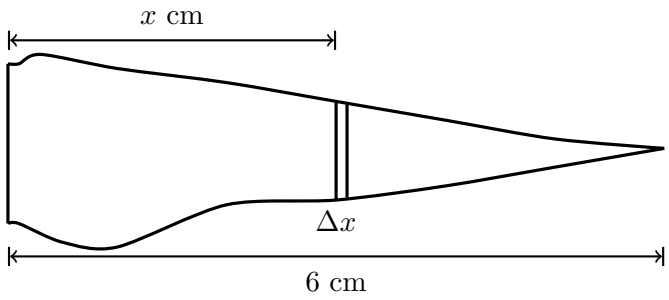
7. [12 points] Hannah Haire has a carrot that is 6 cm long. Lying on its side, it looks like the diagram below, and cross-sections perpendicular to the x-axis are circles. The density of the carrot also varies with x.



Given a distance x cm from the large end of the carrot, let $f(x)$ model the diameter, in cm, of the circular cross-section and $\delta(x)$ the density of the carrot, in g/cm^3.

a. [4 points] Write an expression that gives the approximate mass, in grams, of a slice of the carrot that is Δx cm thick and x cm from the large end of the carrot. (Assume here that Δx is small but positive.) Your expression should not involve any integrals, but may include $f(x)$ and $\delta(x)$.

Answer: $\pi\left(\frac{f(x)}{2}\right)^2 \delta(x)\Delta x$

b. [3 points] Write an expression involving one or more integrals that gives the total mass of the carrot. Your answer may include $f(x)$ and $\delta(x)$.

Answer: $\pi\int_0^6 \left(\frac{f(x)}{2}\right)^2 \delta(x)\,dx$

c. [5 points] Below is a table with some values of $f(x)$ and $\delta(x)$. Use MID(3) to estimate the mass, in grams, of the carrot. Write out every term in your sum.

x	0	1	2	3	4	5	6
$f(x)$	3.4	3.8	2.6	2.1	1.4	0.6	0
$\delta(x)$	1.54	1.52	1.48	1.44	1.42	1.39	1.32

Answer: $2\pi((\frac{3.8}{2})^2 \cdot 1.52 + (\frac{2.1}{2})^2 1.44 + (\frac{0.6}{2})^2 1.39) \approx 45.3$