7. [12 points] Hannah Haire has a carrot that is 6 cm long. Lying on its side, it looks like the diagram below, and cross-sections perpendicular to the x-axis are circles. The density of the carrot also varies with \( x \).

Given a distance \( x \) cm from the large end of the carrot, let \( f(x) \) model the diameter, in cm, of the circular cross-section and \( \delta(x) \) the density of the carrot, in g/cm\(^3\).

a. [4 points] Write an expression that gives the approximate mass, in grams, of a slice of the carrot that is \( \Delta x \) cm thick and \( x \) cm from the large end of the carrot. (Assume here that \( \Delta x \) is small but positive.) Your expression should not involve any integrals, but may include \( f(x) \) and \( \delta(x) \).

Answer: 
\[
\pi \left( \frac{f(x)}{2} \right)^2 \delta(x) \Delta x
\]

b. [3 points] Write an expression involving one or more integrals that gives the total mass of the carrot. Your answer may include \( f(x) \) and \( \delta(x) \).

Answer: 
\[
\pi \int_0^6 \left( \frac{f(x)}{2} \right)^2 \delta(x) \, dx
\]

c. [5 points] Below is a table with some values of \( f(x) \) and \( \delta(x) \). Use MID(3) to estimate the mass, in grams, of the carrot. Write out every term in your sum.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>3.4</td>
<td>3.8</td>
<td>2.6</td>
<td>2.1</td>
<td>1.4</td>
<td>0.6</td>
<td>0</td>
</tr>
<tr>
<td>( \delta(x) )</td>
<td>1.54</td>
<td>1.52</td>
<td>1.48</td>
<td>1.44</td>
<td>1.42</td>
<td>1.39</td>
<td>1.32</td>
</tr>
</tbody>
</table>

Answer: 
\[
2\pi \left( \frac{(3.8)^2}{2} \cdot 1.52 + \left( \frac{2.1}{2} \right)^2 1.44 + \left( \frac{0.6}{2} \right)^2 1.39 \right) \approx 45.3
\]