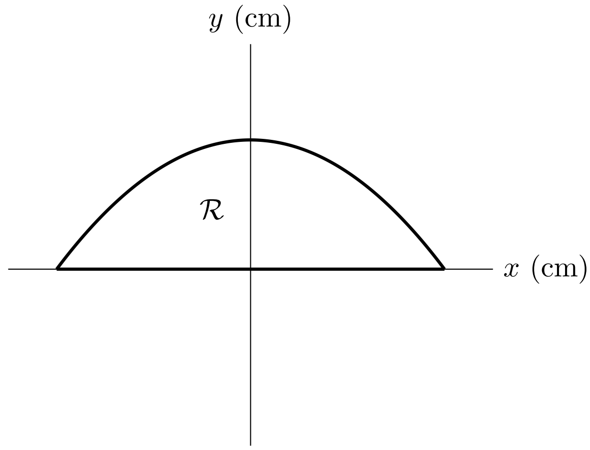
8. [12 points] Consider the region $\mathcal{R}$ bounded by the curve $x^2 + 3y = 4$ and the x-axis.



a. [4 points] Write an expression involving one or more integrals that gives the perimeter, in cm, of $\mathcal{R}$. You do not need to evaluate the integral.

Solution: The parabola intersects the x-axis at $x = \pm 2$. Moreover, we find $y = \frac{1}{3}(4 - x^2)$ so $\frac{dy}{dx} = \frac{2}{3}x$. We plug this into the arc length formula and add the length along the bottom.

Answer: $\int_{-2}^{2} \sqrt{1 + \frac{4}{9}x^2}\, dx + 4$

b. [4 points] Write an expression involving one or more integrals that gives the volume, in cm^3, of the solid formed by rotating $\mathcal{R}$ about the line $x = -4$.

Solution: Using shell method:

$$\int_{-2}^{2} 2\pi(x - (-4))\frac{1}{3}(4 - x^2)\, dx \text{ cm}^3 = \frac{2\pi}{3}\int_{-2}^{2} (x+4)(4 - x^2)\, dx \text{ cm}^3.$$

Using washer method:

$$\int_{0}^{4/3} \pi\left(\sqrt{4 - 3y} - (-4)\right)^2 - \pi\left(-\sqrt{4 - 3y} - (-4)\right)^2\, dy$$

Answer: **(see above)**

c. [4 points] Write, but do not evaluate, an expression involving one or more integrals that gives the mass, in grams, of a thin plate in the shape of the region R that has mass density given by $\delta(x) = x + 2$ g/cm^2.

Answer: $\int_{-2}^{2} \frac{1}{3}(4 - x^2)(x + 2)\, dx$