**10**. [7 points] Consider functions f and g that satisfy all of the following:

- $f(x)$ is defined, positive, and continuous for all $x > 1$.

- $\lim_{x \to 1^+} f(x) = \infty$ (so $f(x)$ has a vertical asymptote at $x = 1$).

- $g(x)$ is defined and differentiable for all real numbers x, and $g'(x)$ is continuous.

- $\dfrac{d}{dx} \left(\dfrac{g(x)}{\ln x} \right) = f(x)$ for all $x > 1$.

- The tangent line to $g(x)$ at $x = 1$ is given by the equation $y = \dfrac{1}{2}(x - 1)$. Graphs of $g(x)$ (solid) and this tangent line (dashed) are shown below.
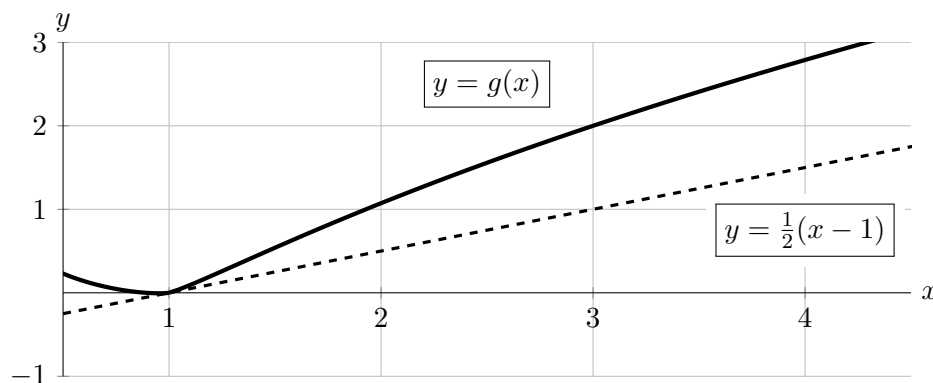
Determine whether the integral $\displaystyle\int_1^3 f(x)\, dx$ converges or diverges.

- If the integral converges, circle "Converges", find its exact value, and write the exact value on the answer blank provided.

- If the integral diverges, circle "Diverges" and carefully justify your answer.

Show every step of your work carefully, and make sure that you use correct notation.

Circle one:

$\displaystyle\int_1^3 f(x)\, dx$ **converges to** _____   or $\displaystyle\int_1^3 f(x)\, dx$ **diverges**