1. [12 points] The two continuous functions $g(x)$ and $h(x)$ have the following properties.

- $\int_{-2}^{4} g(t)\ dt = 11$
- $\int_{-1}^{2} g(t)\ dt = 5$
- $\int_{-0.5}^{1} g(t)\ dt = 3$
- $g(x) = 7$ on the interval $[4, 6]$
- $H(5) - H(1) = -3$, where $H(x)$ is an antiderivative of $h(x)$
- $\int_{-2}^{2} h(x)\ dx = 6.$
- $\int_{5}^{14} h(x)\ dx = 15$
- $\int_{-\infty}^{2} h(x)\ dx = 20.$

Calculate the following values. Write "DIVERGES" if appropriate.
If there is not enough information provided to find the exact value, write "NOT ENOUGH INFO."
You are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown.

a. [3 points] $\int_{-2}^{6} g(x)\ dx$

Solution:

$$\int_{-2}^{6} g(x)\ dx = \int_{-2}^{4} g(x)\ dx + \int_{4}^{6} g(x)\ dx = 11 + \int_{4}^{6} 7\ dx = 11 + 2(7) = 25$$

Answer: 25

b. [3 points] $\int_{-1}^{2} g(2x)\ dx$

Solution: Using substitution with $w = 2x$ (so $dw = 2\ dx$), we find

$$\int_{-1}^{2} g(2x)\ dx = \frac{1}{2}\int_{-2}^{4} g(w)\ dw = \frac{11}{2} = 5.5$$

Answer: 5.5

c. [3 points] The average value of $h(x)$ on the interval $[1, 5]$

Solution: With H as in the bullet points above, the formula for average value of a function gives

$$\frac{1}{4}\int_{1}^{5} h(x)\ dx = \frac{1}{4}\left(H(5) - H(1)\right) = \frac{-3}{4} = -0.75$$

Answer: -0.75

d. [3 points] $\int_{-2}^{\infty} h(-x)\ dx$

Solution:

$$\begin{aligned}\int_{-2}^{\infty} h(-x)\ dx &= \lim_{b\to\infty}\int_{-2}^{b} h(-x)\ dx \\ &= \lim_{b\to\infty}\int_{2}^{-b} -h(w)\ dw \qquad \text{(substitution with } w = -x\text{)} \\ &= \lim_{b\to\infty}\int_{-b}^{2} h(w)\ dw \\ &= \int_{-\infty}^{2} h(w)\ dw = 20\end{aligned}$$

Answer: 20