

4. [5 points] Determine whether the integral $\int_0^3 \frac{1}{x^{\pi/4}} dx$ converges or diverges.

- If the integral converges, circle “Converges”, find its exact value, and write the exact value on the answer blank provided.
- If the integral diverges, circle “Diverges” and carefully justify your answer.

In either case, you must show all your work and use proper notation. Evaluation of integrals must be done **without using a calculator**.

Note that $\frac{1}{x^{\pi/4}} = x^{-\pi/4}$.

Circle one:

$$\int_0^3 \frac{1}{x^{\pi/4}} dx \quad \text{converges to} \quad \frac{3^{-(\pi/4)+1}}{-(\pi/4)+1} \quad \text{or} \quad \int_0^3 \frac{1}{x^{\pi/4}} dx \quad \text{diverges}$$

Solution:

$$\begin{aligned} \int_0^3 \frac{1}{x^{\pi/4}} dx &= \lim_{a \rightarrow 0^+} \int_a^3 \frac{1}{x^{\pi/4}} dx = \lim_{a \rightarrow 0^+} \int_a^3 x^{-\pi/4} dx \\ &= \lim_{a \rightarrow 0^+} \left. \frac{x^{-(\pi/4)+1}}{-(\pi/4)+1} \right|_{x=a}^{x=3} \\ &= \lim_{a \rightarrow 0^+} \left[\frac{3^{-(\pi/4)+1}}{-(\pi/4)+1} - \frac{a^{-(\pi/4)+1}}{-(\pi/4)+1} \right] \\ &= \frac{3^{-(\pi/4)+1}}{-(\pi/4)+1} \quad (\text{since } -(\pi/4)+1 \text{ is positive}) \end{aligned}$$