7. [6 points] A new edition of an old video game features a rocket which blasts off into space along a staight-line path. As in the earlier edition of the video game, at $t = 10$ seconds after the rocket engines ignite, the rocket detaches from the platform and lifts off. The game designers slightly altered the speed function in the new edition to

$$r(t) = \frac{1}{t^{4/5}(1+t^{2/5})}\text{km/second},$$

where t is measured in seconds after the engines ignite so the formula for $r(t)$ given above is valid for $t \geq 10$.

a. Assuming time in the video game goes on forever, write an expression involving an integral that represents the distance from the launchpad that the rocket approaches as time goes on.

Solution:

$$\int_{10}^{\infty} \frac{1}{t^{4/5}(1+t^{2/5})}\,dt$$

Answer: $\int_{10}^{\infty} \frac{1}{t^{4/5}(1+t^{2/5})}\,dt$

b. Determine whether the answer to part **a.** converges or diverges.

- If the integral converges, circle "Converges", find its exact value, and write the exact value on the answer blank provided.
- If the integral diverges, circle "Diverges" and carefully justify your answer.

In either case, you must show all your work and use proper notation. Evaluation of integrals must be done **without using a calculator**.

Hint: let $w = t^{1/5}$.

Circle one: **Converges to** $\frac{5\pi}{2} - 5\arctan(10^{1/5})$ or **Diverges**

Solution: We have

$$\int_{10}^{\infty} \frac{1}{t^{4/5}(1+t^{2/5})}\,dt =$$

$$= \lim_{b\to\infty} \int_{10}^{b} \frac{1}{t^{4/5}(1+t^{2/5})}\,dt.$$

We now substitute with $w = t^{1/5}$ and $dw = \frac{1}{5}t^{-4/5}\,dt$ (or equivalently $5\,dw = \frac{1}{t^{4/5}}\,dt$).

$$= \lim_{b\to\infty} \int_{10^{1/5}}^{b^{1/5}} \frac{5}{1+w^2}\,dw$$

$$= \lim_{b\to\infty} 5\arctan w \Big|_{10^{1/5}}^{b^{1/5}}$$

$$= \lim_{b\to\infty} \left(5\arctan(b^{1/5}) - 5\arctan(10^{1/5})\right)$$

$$\frac{5\pi}{2} - 5\arctan(10^{1/5})$$