8. [12 points] Use the following graph and table to calculate the integrals below.

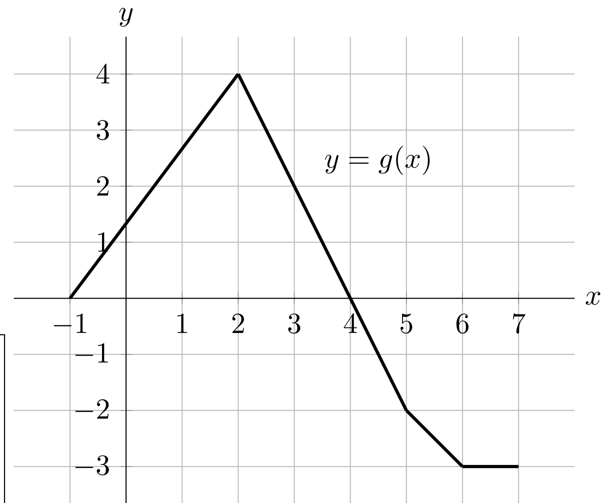
The table below gives several values of a differentiable function f and its derivative f'. Assume that both $f(x)$ and $f'(x)$ are positive and continuous.

x	-2	-1	0	1	3
$f(x)$	0.5	3	4	10	30
$f'(x)$	2	0.5	5	2	22

You are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown.

For each of parts **a.-c.** below, find the exact value of the given quantity. If there is not enough information provided to find the exact value, write "NOT ENOUGH INFO."
All your answers must be in **exact** form.

Let g be the piecewise linear function with graph shown below.



a. [4 points] Find $\displaystyle\int_3^4 tg'(t)\ dt$.

Solution: Use integration by parts, with $u = t$ and $v' = g'(t)$, so $u' = 1$ and $v = g(t)$:

$$\int_3^4 tg'(t)\ dt = tg(t)|_3^4 - \int_3^4 g(t)\ dt$$

$$= 4g(4) - 3g(3) - 1$$

$$= 0 - 6 - 1$$

$$= -7.$$

Alternatively, noting that $g'(t) = -2$ on the interval $[3, 4]$, we have

$$\int_3^4 tg'(t)\ dt = -2\int_3^4 t\ dt = -t^2\Big|_3^4 = -16 + 9 = -7.$$

Answer: -7

b. [4 points] Find $\displaystyle\int_{-1}^1 \frac{2f'(2x+1)}{f(2x+1)}\ dx$.

Solution: Using the substitution $w = f(2x+1)$, so $dw = 2f'(2x+1)$:

$$\int_3^{30} \frac{1}{w}\ dw = \ln|w|\Big|_3^{30}$$

$$= \ln|30| - \ln|3|$$

Answer: $\ln|30| - \ln|3| = \ln|10|$

c. [4 points] Find $\displaystyle\int_1^3 \frac{f'(x)(7f(x)+11)}{(f(x)+1)(2f(x)+4)}\,dx$.

Solution: Starting with the substitution $w = f(x)$, so that $dw = f'(x)\,dx$, the integral becomes

$$\int_{10}^{30} \frac{7w+11}{(w+1)(2w+4)}\,dw$$

We now perform a partial fraction decomposition:

$$\frac{7w+11}{(w+1)(2w+4)} = \frac{A}{w+1} + \frac{B}{2w+4}$$

$$7w + 11 = A(2w+4) + B(w+1)$$

Letting $w = -1$ and $w = -2$, we find that

$$4 = 2A \text{ (so } A = 2) \text{ and } -3 = -B \text{ (so } B = 3).$$

Consequently,

$$\int_{10}^{30} \frac{7w+11}{(w+1)(2w+4)}\,dw = \int_{10}^{30} \frac{2}{w+1} + \frac{3}{2w+4}\,dw$$

$$= \left(2\ln|w+1| + \frac{3}{2}\ln|2w+4|\right)\Bigg|_{10}^{30}$$

$$= 2\ln|31| + \frac{3}{2}\ln|64| - 2\ln|11| - \frac{3}{2}\ln|24|$$

Answer: $2\ln(31) + \frac{3}{2}\ln(64) - 2\ln(11) - \frac{3}{2}\ln(24) = 2\ln(31/11) + \frac{3}{2}\ln(8/3)$