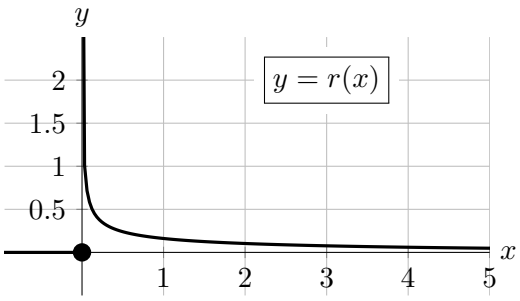
9. [10 points]

It has been suggested that the probability density function given by

$$r(x) = \begin{cases} 0 & \text{if} \quad x \le 0 \\ \dfrac{e^{-0.1x}}{\sqrt{10\pi x}} & \text{if} \quad x > 0 \end{cases}$$

models the size of rainfalls. That is, on a given rainy day, this pdf models the amount x (measured in millimeters) of rain that falls.

A graph of $y = r(x)$ is shown below.



Note that even though $r(x)$ has a vertical asymptote as $x \to 0^+$, it is still a valid pdf.

a. [1 point] Use the formula above and the fact that $r(x)$ is a pdf to find the value of $\displaystyle\int_0^\infty r(x)\,dx$. (You do not need to show any work.)

Answer: $\displaystyle\int_0^\infty r(x)\,dx =$ 1

b. [4 points] Write out all the terms of a MID(4) approximation to the integral $\displaystyle\int_3^5 r(x)\,dx$. Do not evaluate the sum, but the letters r and x should not appear in your answer.

Solution: With 4 subdivisions, we have $\Delta x = \frac{5-3}{4} = 0.5$. Our four midpoints are at $x = 3.25, x = 3.75, x = 4.25$, and $x = 4.75$. Hence our sum is

$$\frac{e^{(-0.1)(3.25)}}{\sqrt{(10\pi)(3.25)}}0.5 + \frac{e^{(-0.1)(3.75)}}{\sqrt{(10\pi)(3.75)}}0.5 + \frac{e^{(-0.1)(4.25)}}{\sqrt{(10\pi)(4.25)}}0.5 + \frac{e^{(-0.1)(4.75)}}{\sqrt{(10\pi)(4.75)}}0.5$$

or

$$0.5\left(\frac{e^{-0.325}}{\sqrt{32.5\pi}} + \frac{e^{-0.375}}{\sqrt{37.5\pi}} + \frac{e^{-0.425}}{\sqrt{42.5\pi}} + \frac{e^{-0.475}}{\sqrt{47.5\pi}}\right)$$

c. [2 points] Is the answer to part **b.** an overestimate or underestimate of $\displaystyle\int_3^5 r(x)\,dx$? Circle your choice below. You do not need to explain.

Circle one: OVERESTIMATE [UNDERESTIMATE] NOT ENOUGH INFORMATION

d. [3 points] Let $q(x)$ be the cumulative distribution function for $r(x)$. Which of the following expressions give the fraction of rainfalls that result in between 2 and 4 millimeters of rain? *Circle* ALL *correct answers.*

i. $r(4) - r(2)$ ii. $r'(4) - r'(2)$ iii. [$q(4) - q(2)$]

iv. $q'(4) - q'(2)$ v. [$\displaystyle\int_2^4 r(x)\,dx$] vi. $\displaystyle\int_2^4 r'(x)\,dx$

vii. $\displaystyle\int_2^4 q(x)\,dx$ viii. [$\displaystyle\int_2^4 q'(x)\,dx$] ix. NONE OF THESE