2. [13 points] Let \( f(x) = \frac{1}{2x^2 + 1} \).

a. [4 points] Approximate the integral \( \int_1^5 f(x) \, dx \) using MID(2). Write out each term in your sum. You do not need to simplify the numbers in your sum, but your final answer should not contain the letter “\( f \)”.

\[
\text{Solution:} \\
\text{MID}(2) = 2 \left( \frac{1}{2(2)^2 + 1} + \frac{1}{2(4)^2 + 1} \right)
\]

b. [4 points] Approximate the integral \( \int_1^5 f(x) \, dx \) using TRAP(2). You do not need to simplify the numbers in your sum, but your final answer should not contain the letter “\( f \)”.

\[
\text{Solution:} \\
\text{TRAP}(2) = 2 \cdot \frac{1}{2} \left( \frac{1}{2(1)^2 + 1} + \frac{2}{2(3)^2 + 1} + \frac{1}{2(5)^2 + 1} \right)
\]

c. [5 points] Compute the exact value for \( \int_1^5 f(x) \, dx \). Show all your steps. You do not need to simplify the numbers in your final answer.

\[
\text{Solution:} \quad \text{Let } x = \frac{1}{\sqrt{2}} \tan \theta. \text{ Then}
\]
\[
\int_1^5 \frac{1}{2x^2 + 1} \, dx = \int_{\tan^{-1}(5\sqrt{2})}^{\tan^{-1}(\sqrt{2})} \frac{1}{1 + \tan^2 \theta} \frac{1}{\sqrt{2}} \sec^2 \theta \, d\theta
\]
\[
= \frac{1}{\sqrt{2}} \int_{\tan^{-1}(\sqrt{2})}^{\tan^{-1}(5\sqrt{2})} \, d\theta
\]
\[
= \frac{1}{\sqrt{2}} (\tan^{-1}(5\sqrt{2}) - \tan^{-1}(\sqrt{2})).
\]