2. [13 points] Let $f(x) = \dfrac{1}{2x^2+1}$.

a. [4 points] Approximate the integral $\int_1^5 f(x)\,dx$ using MID(2). Write out each term in your sum. You do not need to simplify the numbers in your sum, but your final answer should not contain the letter "f".

Solution:

$$\text{MID}(2) = 2 \cdot \left(\frac{1}{2(2)^2+1} + \frac{1}{2(4)^2+1}\right)$$

b. [4 points] Approximate the integral $\int_1^5 f(x)\,dx$ using TRAP(2). You do not need to simplify the numbers in your sum, but your final answer should not contain the letter "f".

Solution:

$$\text{TRAP}(2) = 2 \cdot \frac{1}{2}\left(\frac{1}{2(1)^2+1} + \frac{2}{2(3)^2+1} + \frac{1}{2(5)^2+1}\right)$$

c. [5 points] Compute the exact value for $\int_1^5 f(x)\,dx$. Show all your steps. You do not need to simplify the numbers in your final answer.

Solution: Let $x = \frac{1}{\sqrt{2}}\tan\theta$. Then

$$\begin{aligned}\int_1^5 \frac{1}{2x^2+1}\,dx &= \int_{\tan^{-1}\sqrt{2}}^{\tan^{-1}(5\sqrt{2})} \frac{1}{1+\tan^2\theta}\frac{1}{\sqrt{2}}\sec^2\theta\,d\theta \\ &= \frac{1}{\sqrt{2}}\int_{\tan^{-1}\sqrt{2}}^{\tan^{-1}(5\sqrt{2})} d\theta \\ &= \frac{1}{\sqrt{2}}(\tan^{-1}(5\sqrt{2}) - \tan^{-1}\sqrt{2}).\end{aligned}$$