4. [10 points] After walking in the woods, Flora is making juice with the fruit she picked up at
the next hour. The volume of juice in the jar (in gallons) $t$ minutes after she starts making
juice is given by the function

$$F(t) = \int_{\sin t}^{2t} \frac{50}{100 - \ln(x + 2)} \, dx.$$ 

a. [3 points] Calculate $F'(t)$.

**Solution:**

$$F'(t) = \frac{50}{100 - \ln(2t + 2)} \cdot \frac{2}{2} - \frac{50}{100 - \ln(\sin t + 2)} \cdot \cos t.$$ 

b. [3 points] What is the volume of juice (in gallons) in the jar when Flora starts making the
juice? Briefly explain your answer using the function $F(t)$.

**Solution:**

$$F(0) = \int_{0}^{0} (...) \, dx = 0.$$ 

c. [4 points] Nile wants to know the volume of juice in the jar, yet she is confused by the
function $F(t)$. She knows she can write $F(t)$ using $F'(t)$ and the initial volume of juice in
the jar. Help her by rewriting $F(t)$ in the form

$$F(t) = \int_{a}^{t} \underline{\hphantom{0000}} \, d\underline{\hphantom{0000}} + \underline{\hphantom{0000}}.$$ 

Write the above integral with the blanks filled in, and also give the value of $a$.

**Solution:**

$$F(t) = \int_{0}^{t} \frac{50}{100 - \ln(2x + 2)} \cdot \frac{2}{2} - \frac{50}{100 - \ln(\sin x + 2)} \cdot \cos x \, dx + 0.$$