4. [10 points] After walking in the woods, Flora is making juice with the fruit she picked up at the next hour. The volume of juice in the jar (in gallons) t minutes after she starts making juice is given by the function

$$F(t) = \int_{\sin t}^{2t} \frac{50}{100 - \ln(x+2)}\, dx.$$

a. [3 points] Calculate $F'(t)$.

Solution:

$$F'(t) = \frac{50}{100 - \ln(2t+2)} \cdot 2 - \frac{50}{100 - \ln(\sin t + 2)} \cdot \cos t.$$

b. [3 points] What is the volume of juice (in gallons) in the jar when Flora starts making the juice? Briefly explain your answer using the function $F(t)$.

Solution:

$$F(0) = \int_0^0 (...)dx = 0.$$

c. [4 points] Nile wants to know the volume of juice in the jar, yet she is confused by the function $F(t)$. She knows she can write $F(t)$ using $F'(t)$ and the initial volume of juice in the jar. Help her by rewriting $F(t)$ in the form

$$F(t) = \int_a^t \_\_\_\_\_\, d\_\_\_\_\_ + \_\_\_\_\_.$$

Write the above integral with the blanks filled in, and also give the value of a.

Solution:

$$F(t) = \int_0^t \frac{50}{100 - \ln(2x+2)} \cdot 2 - \frac{50}{100 - \ln(\sin x + 2)} \cdot \cos x dx + 0.$$