5. [10 points] Flora pours herself a cup of juice in a cup with the following shape. The cup is filled to the top.

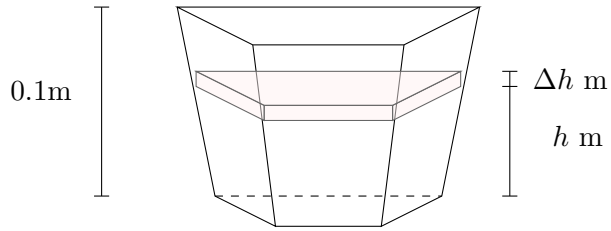

Figure 5.1: Cup

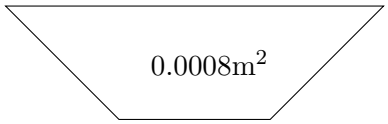

Figure 5.2: Top of cup

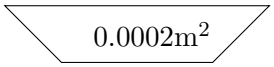

Figure 5.3: Bottom of cup

The area of a horizontal cross section of the cup (as shown in Figure 5.1) is **linear** with respect to h, its height above **the bottom of the cup.**
Flora is going to drink the juice with a magical straw. The top of the straw is always 0.05m above **the top of the cup**. Because the straw is magical, it extends automatically and the bottom end of the straw is always at the surface of the juice. The density of the juice is 1100kg/m^3. The gravitational acceleration is $g = 9.8\text{m/s}^2$.

a. [5 points] What is the approximate mass of the slice of juice that is h meters above **the bottom of the cup**, of thickness Δh meters (as shaded in Figure 5.1)? Do not simplify your answer. Include units.

Solution: First, calculate area of the slice at height h m. This is done by setting up linear equations. Let A be the area of the slice at height h.
When $h = 0$, $A = 0.0002$. When $h = 0.1$, $A = 0.0008$. Thus, slope of the linear equation is $\dfrac{0.0008 - 0.0002}{0.1 - 0} = 0.006$. By point-slope form,

$$A - 0.0002 = 0.006(h - 0),$$

$$A = 0.0002 + 0.006h.$$

The mass of the slice is then

$$\text{mass} = \text{Volume} \cdot \text{density} = (0.0002 + 0.006h)\Delta h \cdot 1100 \text{ kg}.$$

b. [3 points] What is the approximate work needed to lift the same slice of juice (h meters above **the bottom of the cup**, of thickness Δh meters, as shaded in Figure 5.1) to a height of 0.05m above **the top of the cup**? Do not simplify your answer. Include units.

Solution: Distance travelling for the slice is $0.05 + 0.1 - h$ m, so work for slice is

$$\text{Force} \cdot \text{distance travelling} = ((0.0002 + 0.006h)\Delta h \cdot 1100) \cdot g \cdot (0.05 + 0.1 - h) \text{ J}.$$

c. [2 points] Write an expression involving integrals for the total work needed to lift all the juice to a height of 0.05m above **the top of the cup**. Do not evaluate any integrals in your expression. Include units.

Solution:

$$\int_0^{0.1} ((0.0002 + 0.006h) \cdot 1100) \cdot g \cdot (0.05 + 0.1 - h)\, dh \text{ J}.$$