7. [6 points] Split the function $\dfrac{5x^2-7x}{(x-1)^2(x+1)}$ into partial fractions with two or more terms. Do not integrate these terms. Be sure to show all work to obtain your partial fractions.

Solution: Let

$$\frac{5x^2-7x}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}.$$

Solution 1: To solve for B: multiply both sides by $(x-1)^2$, then put $x=1$:

$$\frac{5x^2-7x}{x+1} = A(x-1) + B + C\frac{x-1}{x+1},$$

$$\frac{-2}{2} = 0 + B + 0, B = -1.$$

To solve for C: multiply both sides by $x+1$, then put $x=-1$:

$$\frac{5x^2-7x}{(x-1)^2} = A\frac{x+1}{x-1} + B\frac{x+1}{(x-1)^2} + C,$$

$$\frac{12}{4} = 0 + 0 + C, C = 3.$$

To solve for A, clear the denominator, and compare the coefficients.

$$5x^2-7x = A(x-1)(x+1) - (x+1) + 3(x-1)^2 = (A+3)x^2 - 7x + (2-A).$$

Say we look at the coefficients of x^2:

$$5 = A + 3, A = 2.$$

Therefore,

$$\frac{5x^2-7x}{(x-1)^2(x+1)} = \frac{2}{x-1} - \frac{1}{(x-1)^2} + \frac{3}{x+1}.$$

Solution 2: Clear the denominator.

$$5x^2-7x = A(x-1)(x+1) + B(x+1) + C(x-1)^2 = (A+C)x^2 + (B-2C)x + (-A+B+C).$$

Compare the coefficients.

$$A + C = 5, B - 2C = -7, -A + B + C = 0.$$

Solve the system of equations. An easy way is to write $A = 5 - C$, $B = 2C - 7$, and we then have $-(5-C) + (2C-7) + C = 0$. We have that $A = 2, B = -1, C = 3$.

8. [13 points] Let $f(x)$ be a twice differentiable function with

- $f(0) = 1$.