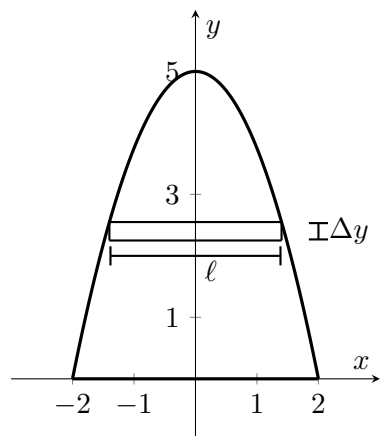
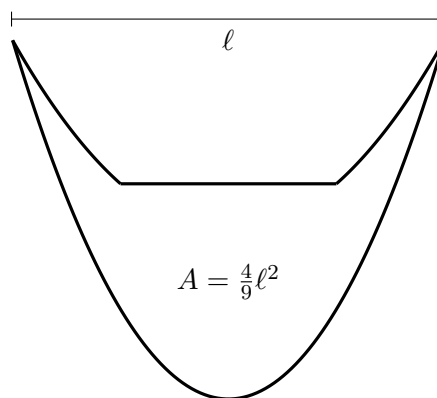


7. [13 points] Brad and Shawna are shipwrecked on an island and are building a new ship out of various materials. The ship has a base given by the region enclosed in the figure on the left, with cross-sections perpendicular to the  $y$ -axis given by the figure on the right. The base is the region bounded by  $y = \frac{-5}{4}(x^2 - 4)$  and  $y = 0$ . The cross-sections have area given by  $\frac{4}{9}\ell^2$  where  $\ell$  is the length of the slice of the base directly below the cross-section. A sample slice of the base of thickness  $\Delta y$  is shown in graph on the left, and all distances are given in meters.



Base of Ship



Cross-section of Ship

- a. [3 points] Write an expression for the length,  $\ell$ , of a the slice  $y$  meters from the  $x$ -axis. Give units.
- b. [3 points] Write an expression for the volume of materials needed to construct a cross-sectional slice of the ship  $y$  meters from the  $x$ -axis with thickness  $\Delta y$  meters. The letter  $\ell$  should not appear in your final answer. Give units.
- c. [3 points] The density of the materials used to make the ship varies. The materials used in the cross section  $y$  meters from the  $x$ -axis is given by  $\delta(y) = (2y + 5) \text{ kg/m}^3$ . What is the mass of a cross sectional slice  $y$  meters from the  $x$ -axis with thickness  $\Delta y$  meters? Give units.
- d. [4 points] Write an integral that gives the total mass of the new boat in  $kg$ . Do not evaluate your integral.