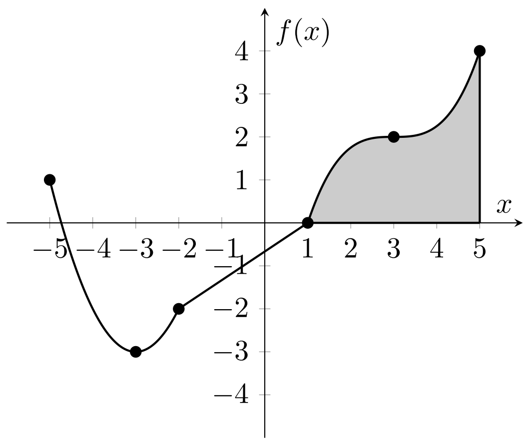
5. [13 points] The function below has a local minimum at $x = -3$, is linear on $[-2, 1]$, and has an inflection point at $x = 3$.



For parts **a.** and **b.**, use the graph of $f(x)$ to determine if the listed quantities are over- or under-estimates for the relevant integral, and write the word OVERESTIMATE or UNDER-ESTIMATE as appropriate. If there is not enough information, write NI.

a. [4 points] $\int_{-3}^{3} f(x)dx$

LEFT(4) **UNDER** RIGHT(4) **OVER**

MID(4) **N.I.** TRAP(4) **N.I.**

b. [4 points]
$\int_{-5}^{1} f(x)dx$

LEFT(12) **N.I.** RIGHT(12) **N.I.**

MID(12) **UNDER** TRAP(12) **OVER**

c. [5 points] The function $f(x)$ on $[1, 5]$ is given by the formula $\frac{1}{4}(x-3)^3 + 2$. Write, but do not solve, an integral giving the volume of the shaded region rotated around $y = -2$.

Solution: Washer Method: The inner radius is given by $r_{in}(x) = 2$ and the outer radius is by $r_{out}(x) = \frac{1}{4}(x-3)^3 + 4$.
Washer formula give:

$$\int_1^5 \pi \left(\left(\frac{1}{4}(x-3)^3 + 4 \right)^2 - 2^2 \right) dx$$

Shell Method: Invert the function so we get a function of y, $f(y) = (4(y-2))^{\frac{1}{3}} + 3$. Then our shells have height $5 - f(y)$ with radius $(2+y)$. Evalutating with the shell formula we have:

$$\int_0^4 2\pi(2+y)\left(5 - (4(y-2))^{\frac{1}{3}} + 3\right) dy$$