6. [13 points]

a. [4 points] Which, if any, of the following are antiderivatives of the function e^{x^2}? Circle ALL that apply, or 'NONE' as appropriate.

$\displaystyle\int_1^{x^2} \frac{e^t}{2t} dt$ $\boxed{\displaystyle\int_{\sqrt{2}}^{x} e^{t^2} dt}$ $\displaystyle\frac{e^{x^2}}{2x}$

$\displaystyle\int_0^{x^2} e^t dt$ NONE

b. [4 points] Back in his younger days, Brad hiked to the top of Mount Olympus. Let the continuous function $T(m)$ denote the rate of change of temperature, in degrees Celsius per meter, after Brad has hiked m meters. Suppose the following mathematical statements hold:

- $\displaystyle\int_0^{2500} T(m)dm = -25.$
- $\displaystyle -0.01 = \frac{1}{1000}\int_0^{1000} T(m)dm.$

Which of the following statements MUST be true? Circle ALL that apply, or 'NONE OF THE ABOVE' as appropriate.

i) $T(m)$ is negative for all values of m in its domain.

ii) The average rate of change of temperature per meter hiked was the same during the first 1000 meters Brad hiked as it was in the next 1500 meters he hiked.

iii) During the first 1000 minutes Brad was hiking, the temperature decreased by an average of 0.1 degrees Celsius per minute.

iv) The temperature decreased by 10 degrees Celsius during the first 1000 meters of Brad's hike.

v) NONE OF THE ABOVE

c. [5 points] What is the mass of a solid cube with side length ℓ centimeters if its density x centimeters above its base is $x + 1$ grams per cubic centimeter for $0 \leq x \leq \ell$? Show all your work including evaluating all integrals and give your answer in terms of ℓ.

Solution: The integral is given by:

$$\int_0^{\ell} \ell^2(x+1)dx = \ell^2\left(\frac{x^2}{2}+x\right)\Big|_0^{\ell} = \frac{\ell^4}{2}+\ell^3$$